

# THE MEDIAN SOLUTION OF THE NEWSVENDOR PROBLEM AND SOME OBSERVATIONS

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ABSTRACT

We consider the median solution of the Newsvendor Problem. Some properties of such a solution are shown through a theoretical analysis and a numerical experiment. Sometimes, though not often, median solution may be better than solutions maximizing expected profit, or maximizing minimum possible, over distribution with the same average and standard deviation, expected profit, according to some criteria. We discuss the practical suitability of the objective function set and the solution derived, for the Newsvendor Problem, and other such random optimization problems.

KEYWORDS

newsvendor problem, median solution, practical suitability of solutions.

## Introduction

The newsvendor problem (see, e.g., [1, 2]) is a fundamental problem in inventory management theory (it is also known by the names as, the newsboy problem, the Christmas tree problem, the single-period problem). In the classical newsvendor problem (NVP), a newsvendor (NV) buys a number of units of a newspaper to meet the demand of the day. The demand is probabilistic. It is a random variable with a known distribution function. Each unit is bought at a purchasing price and sold at a selling price, which gives a profit per unit. Some units may not be sold. These are disposed off at a salvage price per unit. Known such information, it has to be decided how many units of the newspaper are to be stocked by the NV. Often this is done maximizing the expected profit (equivalently, minimizing the expected cost, considering a cost, per unit, due to understocking, and a cost, per unit, due to overstocking). Despite a simple set up, the model has applications in inventory planning, supply chain modeling, revenue management, insurance, etc.

Arrow et al. [3] first used the model in the context of inventory planning and derived the impor-

tant fractile solution, maximizing the expected profit. We shall refer to this solution as maximum expected profit solution (MEPS) in the succeeding. The literature on the NVP is extensive. Many models are available of the problem and many variations and extensions have been proposed of it. In an early work (published in 1958), reproduced later, Scarf [4] has considered that, the demand distribution is not known, but the average of the demand and its variance are. A solution (referred to as Scarf's solution in the later part in this article), maximizing the minimum expected profit possible for such distributions having the average and variance, is derived by him. The solution is as:

$$Q = \begin{cases} \mu \left( 1 + (1 - 2c) \sqrt{\frac{\gamma - 1}{4c(1 - c)}} \right), & \text{if } \gamma c \leq 1; \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

where  $\mu$  is average demand,  $c$  is purchasing price per unit,  $\gamma = (\sigma^2 + \mu^2)/\mu^2$ ,  $\sigma^2$  is the variance of the demand. Selling price is 1 per unit, and there is no salvage price. The analysis also considers no stocking of the item (what may be done in other methods also using some criterion). If the variance of the demand is too high, or for the prices, it may be better not to be in the business at all, incurring no cost. In

subsequent expositions, alternative objective functions as, probability that the profit exceeds a targeted amount, expected profit minus the variance/standard deviation, multiplied with a positive coefficient, have been considered for the NVP. More general situations have been taken into account, extending the problem. These include, different pricing policies such as when discounts are available in purchasing price of the item, stock-dependent demand/ other parameters (e.g., [5, 6]), multiple items (e.g., [7–9]), multiple periods, multi-echelon systems, etc. Some other references on such models are found in [1].

The practically suitable interpretation of probability in the context of NVP, is the long term relative frequency interpretation. For instance, if the expected cost is minimized, we may see it as the solution that minimizes average cost, over a large number of trials, of the underlying random experiment. However, we need a large number of trials, theoretically tending to infinity, here. This creates doubt in the mind of the practicing managers and it is indeed debatable. The problem is further complicated in the presence of dynamic conditions. Hardly conditions will remain the same over time, when we consider a large number of trials. We discuss the median solution in this context. Suppose, there are two solutions –  $A$  &  $B$ . If the probability that solution  $A$  yields less cost than that of solution  $B$  is high, say 0.99, when would it be reasonable to choose  $A$  over  $B$ ? If 0.99 proportion of times  $A$  gives less cost than what  $B$  gives, it is very much anticipated that so would be the case in a single trial. However, we may not be concerned with a single trial. Again, considering a very large number of trials may not be appropriate. Then, what should be one's course of action? We discuss such issues further in the succeeding.

The rest of the article is organized in this manner. In the next section, we give a review of the literature of some of the NVP variants. In the succeeding section, we write the analysis to consider the median as the solution for the NVP, to increase the probability of incurring less cost. We report about a numerical experiment about the effectiveness, considering another suitable criterion, of the solution, here. Thereafter we conclude with some relevant remarks. Here we extend our conclusions to other random optimization problems too.

## **NVP models differing in the objective function**

We give a review of the different models of the NVP that have been discussed in the literature. We confine ourselves mostly to some of such models, in

which only the objective function is changed, but otherwise the same set up as in the classical NVP is maintained, including the cost structure. It is clear that, demand being random, we cannot consider maximizing profit itself, but some parameter as expected profit, etc., have to be the objective. We already have discussed MEPS and Scarf's solution.

Kabak and Schiff [10] have considered the objective of maximizing the probability of attaining a targeted profit. They have given a closed-form solution, an optimal one, when the demand is exponentially-distributed. Lau [11] has considered the same objective (and also some other objectives). He has given a closed-form optimal solution for normally-distributed demand, and numerical methods to obtain such a solution for beta, gamma and Weibull distributions. The author, further, has considered a linear combination of expected profit and the standard deviation of profit. The solution needs to be obtained numerically.

Sometimes not the profit itself, but a utility function of the profit has been considered. The expected utility, or some other parameter derived from the utility, is considered as the objective function. Among such authors are Lau [11], Eeckhoudt [12] Keren and Pliskin [13]. Wang and Webster [14], Wang et al., [15]. These models have sometimes been called as risk-averse, loss-averse, etc., NVP, dependent on the properties of the utility function. Some properties of such solutions, compared to MEPS, have been established.

Consideration of a different objective function, of course, alters the optimal solution. Schweitzer and Cachon [16] give an account how managers may take the decisions in practice, in the context of the NVP. Such decisions seem to deviate from those of the theoretical models discussed in the literature. Experiments as this and such deviations have been reported by other authors also. Su [17] offers an explanation of such deviations writing that decision-makers have bounded rationality and are not perfect optimizers. They do not choose an optimum solution always, but choose solutions with higher utility more frequently. In the model discussed by him, a solution with utility  $u$ , is chosen with probability proportional to  $e^{u/\beta}$ ,  $u$  being the expected profit for the solution,  $\beta$  ( $> 0$ ), a constant denoting the extent of bounded rationality. However, according to our understanding, if bounded rationality arises from psychological bias, unverified heuristics, cognitive constraints etc., that is then only a shortcoming in the analysis. The decision-maker should only be advised to avoid that. But, if the decision-maker has some other objective, or there are some other concerns that have not been

included in the model (e.g., parameters cannot be known precisely), that becomes a fundamental issue.

## Analysis of the median solution for the NVP

We use the following notation. First we would discuss MEPS, in more details.

$X$ : Random variable denoting the demand of the item;

$F(x)$ : Distribution function of  $X$ ;

$f(x)$ : Probability density function (pdf) of  $X$ ;

$\mu$ : Average of  $X$  ( $0 < \mu < \infty$ );

$c$ : Cost of purchasing, per unit;

$r$ : Selling price, per unit;

$s$ : Salvage price, per unit;

$C_O$ : Cost of overstocking the item, per unit of the item;

$C_U$ : Cost of under-stocking the item, per unit of the item;

$Q$ : Quantity of the item to be stocked, decision variable;

$K_1(x)$ : Cost incurred when the demand is  $x$ , with the stock quantity  $Q$ ;

$K_2(x)$ : An equivalent expression of  $K_1(x)$ .

Then the cost (negative of profit) may be written as,

$$K_1(x) = \begin{cases} cQ - rx - s(Q - x), & x \leq Q, \\ cQ - rQ, & x > Q. \end{cases} \quad (2)$$

This is same as,

$$K_1(x) = \begin{cases} (c - s)(Q - x) - (r - c)x, & x \leq Q, \\ (r - c)(x - Q) - (r - c)x, & x > Q. \end{cases} \quad (3)$$

When we calculate the average of  $K_1(x)$ , the term  $(r - c)x$  would give  $(r - c)\mu$ , a constant. So, when minimizing the expected cost, we may consider equivalently,

$$K_2(x) = \begin{cases} C_O(Q - x), & x \leq Q, \\ C_U(x - Q), & x > Q. \end{cases} \quad (4)$$

where  $C_O = (c - s)$ ,  $C_U = (r - c)$ .

The demand is non-negative,  $F(x) = 0$ , for  $x < 0$ . We shall consider that the cost coefficients are positive,  $C_O, C_U > 0$ . With the assumption that, the restriction of  $F(x)$ ,  $x \geq 0$  is invertible,  $f(x)$  is continuous for  $x > 0$ , we have the solution that minimizes the expected cost as,  $Q = F^{-1}(C_U / (C_O + C_U))$ . This is the well-known fractile formula of the NVP. This is valid also when  $0 < F(x) < 1$ ,  $a < x < b$  ( $b > a \geq 0$ ),  $F(a) = 0$ ,  $F(b) = 1$  and  $F(x)$  is invertible in  $(a, b)$ ,  $f(x)$  is continuous in  $(a, b)$ . We have an analogous solution when  $F(x)$  is discrete.

In the succeeding, we, first, give an analysis to derive some properties of the median solution. Then we have the results of a numerical experiment on another criterion, for which a theoretical analysis is not tractable.

## Analysis

We have the same model, as in the preceding, of the NVP. But we attempt to obtain a solution such that, compared with any other solution, the probability that it yields less or equal cost is relatively high. We consider, for simplicity of analysis, that,  $F(x)$  is continuous everywhere and the median is defined uniquely for  $F(x)$ . (Otherwise also, same kind of analysis can be done.)

We consider the cost as in (2) or (3). But when we compare the costs of some solutions ( $Q$  values) we may consider (4) equivalently as  $(r - c)x$  would be the same for all such values. For any realization, the cost (4) would be zero, if  $Q = x$ . For  $Q > x$ , the cost would increase linearly with  $(Q - x)$ , with the rate of  $C_O$ ; for  $Q < x$ , cost increases linearly with  $(x - Q)$ , with the rate of  $C_U$  (see Fig. 1). (In the figures,  $x$ -axis gives demand values of the item;  $y$ -axis gives the pdf value  $f(x)$ , shown by thin line; and the costs  $K_2(x)$ , for different solutions. Costs are shown by bold lines. If the solution is  $Q$ ,  $K_2(x) = 0$ , for  $x = Q$ ).

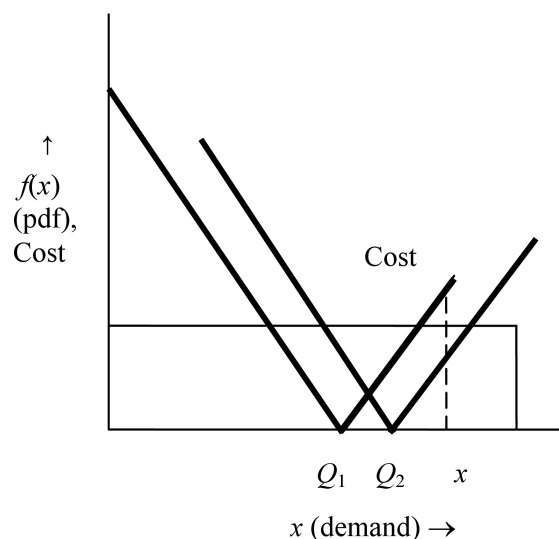


Fig. 1. Cost of solutions in a demand realization.

Therefore, if we choose  $Q = F^{-1}(0.5)$ ,  $Q$  yields less cost than any other solution considered, with probability more than 0.5. No other solution has this property. For any other solution, there would be some solutions which would have probability more than 0.5 to have less cost. It is not difficult to calculate the

probability that a solution would have larger cost than that of median solution. Let,  $S$  be another solution. If  $S < Q$  (median), then the probability that the median solution would have less cost is,  $1.0 - F((C_U S + C_O Q)/(C_U + C_O))$ . If  $S > Q$ , the probability is  $F((C_U Q + C_O S)/(C_U + C_O))$ . See Fig. 2.

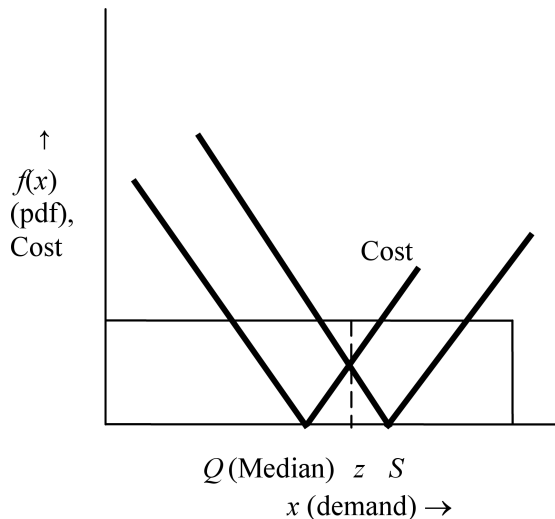


Fig. 2. Median solution is better than solution  $S$ , up to demand  $z$ .

We may also note that, the probability of less cost is higher for the median solution even if the under-stocking & overstocking costs are nonlinear, but strictly increasing.

However, median solution may not be the solution that is the best among some solutions considered, with the highest probability, i.e., most often.

### Numerical examples

(a) Suppose, the demand has a (truncated continuously around 0.0 and 200.0) normal distribution with average 100.0 and standard deviation of 10.0.  $C_O = 1$ ,  $C_U = 5$ . Minimizing the expected cost, we would get the solution of 109.67. The median solution is 100.0. The probability that this solution would have less cost than that of the other is 0.564. Expected cost (with (4)) may be calculated numerically for the first solution as 14.98 and for the median solution as 23.82.

(b) We have the same data as in (a), but the demand follows an exponential distribution with average 100.0. The median of the distribution is 30.10. MEPS is 77.82. So, the median solution has less cost than this solution with probability 0.618. The skewness of the distribution gives somewhat higher probability. Expected cost for the median solution is 417.50 and that of the other is 389.96.

### Numerical experiment

We have conducted a numerical experiment to examine the effectiveness of the median solution with another criterion. It is established in the preceding that, compared to any other solution, the probability that the median solution would have less cost is more than 0.5. In the numerical experiment, we see the probability that this solution would have less cost considering the total of three independent and identically-distributed random variables. For a given solution, each random variable has the distribution, of the cost, that results from the independent demand distributions and the solution. In other words, we consider the total cost of three periods, in which the same (median or other) solution is used.

We consider four demand distributions – uniform distribution in  $[0, 100)$ , symmetric triangular distribution in  $[0, 100)$  and normal distribution (truncated as before) with average (and median) of 50.0 and standard deviation of 15.0 (very nearly), exponential distribution with average 50.0 (hence, standard deviation 50.0). It has median of 15.05. The first three distributions are symmetric, median being same as average. Variance of the uniform distribution is 833.33 and that of the triangular distribution is 416.67. The costs are as,  $C_O = 1$ ;  $C_U = 0.8, 1.2, 3.0$  &  $5.0$  for first three case and  $C_U = 1.2, 3.0$  &  $5.0$  for exponential distribution, giving 15 experimental observations. ( $C_U = 0.8$  is not included for exponential distribution as Scarf's solution would be zero in this case.) For each observation, 30000 trials or realizations are simulated to determine the probabilities. The solutions considered are as,  $Q_1$  (minimum expected cost solution),  $Q_2$  (Scarf's solution),  $Q_3$  (median solution). The empirical results are reported in Table 1. The experiment has been done with MS Excel, using the language of Visual Basic. Random numbers are generated with the Rnd() function.

As we see in the empirical results, the probability that the median solution would have less total cost, the sum of 3 independent random variables, is more than 0.5 in comparison to any one solution, or both the solutions, in the cases when under-stocking and overstocking costs do not differ much (e.g., difference not being more than 20%). It may be said that, higher effectiveness of the solution is seen with the criterion, as considered, for such cases. But, the median solution may be inferior to the MEPS otherwise. Our further experimental observations also suggest that, some perturbation (e.g., a change of 10%) may give a better solution than the MEPS often, according to the criterion considered.

Table 1  
 Observations of the numerical experiment.

Obs. No.	Demand distribution	$C_U$	$Q_1$ (minimum expected cost solution)	$Q_2$ (Scarf's solution)	$Q_3$ (Median)	Probability $\text{Cost}(Q_3) < \text{Cost}(Q_1)$	Probability $\text{Cost}(Q_3) < \text{Cost}(Q_2)$	Probability $\text{Cost}(Q_3) < \text{Cost}(Q_1)$ and $\text{Cost}(Q_3) < \text{Cost}(Q_2)$
1	Uniform [0, 100)	0.8	44.44	46.77	50.0	0.5281	0.5165	0.5165
2		1.2	54.54	52.63	50.0	0.5342	0.5232	0.5232
3		3.0	75.0	66.67	50.0	0.3012	0.2375	0.2375
4		5.0	83.33	75.82	50.0	0.2753	0.2366	0.2366
5	Triangular [0, 100)	0.8	47.14	47.71	50.0	0.5295	0.5241	0.5241
6		1.2	52.33	51.86	50.0	0.5207	0.516	0.516
7		3.0	64.64	61.79	50.0	0.3007	0.2635	0.2635
8		5.0	71.13	68.26	50.0	0.2975	0.2727	0.2727
9	Normal(50, 15) (Truncated)	0.8	47.90	48.32	50.0	0.5318	0.5256	0.5256
10		1.2	51.71	51.37	50.0	0.523	0.5182	0.5182
11		3.0	60.12	58.66	50.0	0.3029	0.2751	0.2751
12		5.0	64.51	63.42	50.0	0.2749	0.261	0.261
13	Exponential(50)	1.2	17.12	54.56	15.05	0.533	0.8721	0.533
14		3.0	30.10	78.87	15.05	0.3123	0.7478	0.3123
15		5.0	38.91	94.72	15.05	0.3256	0.6745	0.3256

## Conclusions

We have discussed the median solution of the newsvendor problem. It has the property that, the probability that the solution gives less cost than that of any other solution is more than 0.5, irrespective of the demand distribution. No other solution has this property. With some values of the parameters, compared with MEPS, this probability may even tend to 1.0. Still then, the question remains if the median solution is more suitable than the MEPS, or other solutions.

If we consider a large number of identical trials, and the total or the average of the costs incurred, MEPS would be better than any other solution, applying the long term frequency interpretation of probability. However, consideration of such a total may be questionable, given the ever-dynamic nature of conditions with time (or that, there may not be so many trials ever to occur). In an attempt to find an answer to this, we see the probability of having less total cost in three periods. This is done with a numerical experiment. We observe that, the probability that the median solution would have less

total cost than that of MEPS is more than 0.5, when under-stocking and overstocking costs are comparable. This holds also in comparison to Scarf's solution. Although, for such cases, MEPS is almost the same as the median solution. For other ranges of the costs, MEPS is clearly better. But there may be solutions better than MEPS.

For random optimization problems (by the term we mean that, the decision problem is modelled with a random variable and some objective function, seen as a parameter of it, has to be set appropriately and optimized), generally, for evaluating the suitability of different types of objectives & solutions, we need to see the practical requirements. Suppose, we need to consider a single realization of the underlying random experiment. Then, for two solutions, we should see for what solution the probability is higher to give less cost (if we are concerned about decreasing cost). Since we would not be adding the costs of many realizations, we should not consider the average cost of a solution. But consideration of the probability, as mentioned, too involves a large number of trials. If we are to see just one realization, not cumulate the costs of many realizations, the higher probabili-

ty gives an intuitive “confidence” to do better than a competitive solution, in just a single realization (we have such interpretations for other probability optimizing solutions too). Median solution is the best solution for the NVP, according to this view.

Analogously, if we need to consider, not one, but a small number of trials of the same the random experiment, then there is another extended random experiment, with as many trials. We see for which solution the probability of having less cost than that of a competitive solution is more, for it. But, again, there is nothing beyond these trials. In actuality, however, we mostly would have, what we may say, a continuation of many random experiments, each with a small number of trials. The analysis of such a situation would require a larger random experiment, involving the preceding random experiments and the required trials thereof. We would have a single trial of this larger random experiment and we may see the “confidence”.

Because “confidence” does not guarantee any performance, there is no exact answer. Though, intuitively, supported by the numerical experiment as reported in this article, MEPS type solutions (possibly with some perturbation, extent of which should decrease with higher number of trials for one random experiment) for each of the smaller random experiment would be satisfactory, when we see total cost. Given a practical problem, the solutions applied should be verified with suitable numerical experiments, as much as possible.

Further research, including numerical experiments, on the issues, as discussed, will be valuable. The issue of imprecise information about model parameters also exists. In the presence of high imprecision, not having any pattern, not much is analyzable, and decision-making becomes extremely difficult.

We hope that, the observations and the discussions in this article should be useful to practicing decision-makers and management researchers.

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## References

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- [1] Khouja M., *The single-period (news-vendor) problem: literature review and suggestions for future research*, *Omega*, 27, 537–553, 1999.
- [2] Qin Y., Wang R., Vakharia A.J., Chen Y., Michelle M.H., Seref M.M.H., *The news-vendor problem: Review and directions for future research*, *European Journal of Operational Research*, 213, 361–374, 2011.
- [3] Arrow K., Harris T., Marshack J., *Optimal Inventory Policy*, *Econometrica*, 19, 250–272, 1951.
- [4] Scarf H., *Inventory theory*, *Operations Research*, 50, 1, 186–191, 2002.
- [5] Dana Jr. J., Petruzzi N., *Note: The newsvendor model with endogenous demand*, *Management Science*, 47, 1488–1497, 2001.
- [6] Cachon G.P., Kok A.G., *Implementation of the Newsvendor Model with Clearance Pricing: How to (and How Not to) Estimate a Salvage Value*, *Manufacturing & Service Operations Management*, 9, 3, 276–290, 2007.
- [7] Van Mieghem J., Rudi N., *Newsvendor networks: inventory management and capacity investments with discretionary activities*, *Manufacturing & Service Operations Management*, 4, 313–335, 2002.
- [8] Zhang B., Hua Z., *A portfolio approach to multi-product newsboy problem with budget constraint*, *Computers & Industrial Engineering*, 58, 4, 759–765, 2010.
- [9] Choi S., Ruszczyński A., *A multi-product risk-averse newsvendor with exponential utility function*, *European Journal of Operational Research*, 214, 78–84, 2011.
- [10] Kabak I., Schiff A., *Inventory models and management objectives*, *Sloan Management Review*, 10, 53–59, 1978.
- [11] Lau H.S., *The newsboy problem under alternative optimization objectives*, *Journal of the Operational Research Society*, 31, 1, 525–535, 1980.
- [12] Eeckhoudt L., Gollier C., Schlesinger H., *The risk averse (and prudent) newsboy*, *Management Science*, 4, 786–794, 1995.
- [13] Keren B., Pliskin J., *A benchmark solution of the risk-averse newsvendor problem*, *European Journal of Operational Research*, 174, 1643–1650, 2006.
- [14] Wang C.X., Webster S., *The loss-averse newsvendor problem*, *Omega*, 37, 93–105, 2009.
- [15] Wang C.X., Webster S., Suresh N.C., *Would a risk-averse newsvendor order less at a higher selling price?*, *European Journal of Operational Research*, 196, 544–553, 2009.
- [16] Schweitzer M.E., Cachon G.P., *Decision bias in the newsvendor problem with a known demand distribution: experimental evidence*, *Management Science*, 46, 404–420, 2000.
- [17] Su X., *Bounded Rationality in Newsvendor Models*, *Manufacturing & Service Operations Management*, 10, 4, 566–589, 2008.