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A COMPREHENSIVE MATHEMATICAL PROGRAMMING MODEL FOR MINIMIZING COSTS IN A MULTIPLE-ITEM REVERSE SUPPLY CHAIN WITH SENSITIVITY ANALYSIS

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ABSTRACT
These instructions give you guidelines for preparing papers for IFAC conferences. A reverse supply chain is configured by a sequence of elements forming a continuous process to treat return-products until they are properly recovered or disposed. The activities in a reverse supply chain include collection, cleaning, disassembly, test and sorting, storage, transport, and recovery operations. This paper presents a mathematical programming model with the objective of minimizing the total costs of reverse supply chain including transportation, fixed opening, operation, maintenance and remanufacturing costs of centers. The proposed model considers the design of a multi-layer, multi-product reverse supply chain that consists of returning, disassembly, processing, recycling, remanufacturing, materials and distribution centers. This integer linear programming model is solved by using Lingo 9 software and the results are reported. Finally, a sensitivity analysis of the proposed model is also presented.

KEYWORDS
reverse supply chain, mathematical modeling, sensitivity analysis.

Introduction

With the increased environmental concerns and stringent environmental laws, companies focus on setting up a reverse supply chain either because of environmental regulations or to reduce their operating costs by reusing products or components. According to the American Reverse Logistics Executive Council, Reverse Logistics is defined as Rogers and Tibben-Lembke [1]: "The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal".

Implementation of reverse logistics would allow not only for cost savings in inventory carrying, transportation, and waste disposal, but also for the im-

provement of customer loyalty and future sales [2, 3]. A group of companies has gone further and achieved economic gains from the adoption of environment-friendly logistic networks. For instance, Nike, the shoe manufacturer encourages consumers to bring their used shoes to the store where they had purchased them. These shoes are then shipped back to Nike's plants and made in to basketball courts and running tracks. By donating the material to the basketball courts and donating funds for building and maintaining these courts, Nike has enhanced the value of its brand [1].

In a broader sense, reverse logistics refers to the distribution activities involved in product returns, source reduction, conservation, recycling, substitution, reuse, disposal, refurbishment, repair and remanufacturing [4]. Reusable parts can be removed from the product, returned to a manufacturer where

they can be reconditioned and assembled into new products [5]. Recycling (with or without disassembly) includes the treatment, recovery, and reprocessing of materials contained in the used products or components in order to replace the virgin materials in the production of new goods [6]. Re-manufacturing is the process of removing specific parts of the waste product for further reuse in new products. Disposal is the processes of incineration or landfill [6].

The rest of the paper is organized as follows. Section 2 presents a brief review of the literature on reverse supply chain. A general framework and problem definition for reverse supply chain are described in Sec. 3. The proposed the mathematical model of the reverse supply chain is given in Sec. 4. Numerical experiments are provided in Sec. 5. In Sec. 6, Sensitivity analysis are presented. Finally conclusions and further researches are addressed in the last section.

Literature review

For the last decade, increasing concerns over environmental degradation and increased opportunities for cost savings or revenues from returned products prompted some researchers to formulate more effective reverse logistics strategies. In remanufacturing models, Kim, Song, and Jeong [7] discussed a notion of remanufacturing systems in reverse logistics environment. They proposed a general framework in view of supply planning and developed a mathematical model to optimize the supply planning function. The model determines the quantity of products parts processed in the remanufacturing facilities subcontractors and the amount of parts purchased from the external suppliers while maximizing the total remanufacturing cost saving. Aras et al. [8] develop a nonlinear model and tabu search solution approach for determining the locations of collection centers and the optimal purchase price of used products in a simple profit maximizing reverse logistics network. Teunter et al. [9] dealt with the question of when companies should use shared resources for production and remanufacturing and when they should use specialized resources. In their study, Zuidwijk and Krikke [10] considered two strategic questions in the context of closed-loop supply chains to establish how much a company should invest in product design and how much in the production processes to process their returned products. They formulated the problem as both an integer linear programming and a rules of thumb-based problem.

Du and Evans [11] minimize tardiness and total costs for location and capacity decisions in a closed-loop logistics network operated by third party logistics (3PL) providers. To solve the bi-objective MILP model, a hybrid scatter search method is developed. Kannan et al. [12] developed a mathematical model for a case of battery recycling. However, they did not consider uncertainty of parameters. Amin and Zhang [13] designed a network based on product life cycle. They utilized mixed-integer linear programming to configure the network. Du and Evans [14] developed a bi-objective model for a reverse logistics network by considering minimization of the overall costs, and the total tardiness of cycle time.

Jayaraman, Patterson, and Rolland [15] propose a general mixed-integer programming model and solution procedure for a reverse distribution problem focused on the strategic level. The model decides whether each remanufacturing facility is open considering the product return flow. Ko and Evans [16] consider a network operated by a 3PL service provider and they present a MINLP model for the simultaneous design of the forward and return network. They develop a genetic algorithm-based heuristic to solve the complex Developed model.

Pati, Vrat, and Kumar [17], they developed an approach based on a mixed integer goal programming model (MIGP) to solve the problem. The model studies the inter-relationship between multiple objectives of a recycled paper distribution network. The objectives considered are reduction in reverse logistics cost. Salema, Povoa, and Novais [18] have proposed a MILP model to analyze the problem of closed loop supply chains. They consider multi-product returns with uncertain behavior but limit their consideration of demand for returned products to factories and not to secondary markets or spare markets. Thus a supplier network which may be required to remanufacture a new product to meet the market demand is not considered. Also, this model is not suitable for modular products.

Sheu et al. [19] formulated a linear multi objective programming model to optimize the operations of both integrated logistics and corresponding used-product reverse logistics in a given green-supply chain. Factors such as the used-product return ratio and corresponding subsidies from governmental organization for reverse logistics were considered in the model formulation. The authors also proposed a real world case study for a Taiwan based notebook computer manufacturer.

Fleischmann et al. [20] extended a forward logistics model to a reverse logistics system and discussed the differences. They utilized mixed-integer linear programming model. Kannan et al. [21] proposed a model using genetic algorithm and particle swarm techniques. They applied the model by con-

sidering two cases including a tyre manufacturer and a plastic goods manufacturer. Shi et al. [22] proposed a mathematical model to maximize the profit of a re-manufacturing system by developing a solution approach based on Lagrangian relaxation method.

Schultmann et al. [23] developed a hybrid method to establish a closed-loop supply chain for spent batteries. The model included a two stage (collection point-sorting – recycling or disposal) facility location optimization problem. The authors found the optimal sorting centers to open to serve the recycling facilities through a mixed integer linear programming model which minimizes the total cost, and implemented the model in GAMS (General Algebraic Modeling System) and solved it using a branch-and-bound algorithm. As a hybrid method, it also approached to a simulation under different scenarios for a steel making process. Listes [24] presented a generic stochastic model for the design of networks comprising both supply and return channels, organized in a closed loop system. The author described a decomposition approach to the model, based on the branch-and-cut procedure known as the integer L-shaped method. Wang and Hsu [25] proposed an interval programming model where the uncertainty has been expressed by fuzzy numbers. Gupta and Evans [26] proposed a non-preemptive goal programming approach to model a closed-loop supply chain network.

Pishvaee et al. [27] considered minimization of the total costs, and maximization of the responsiveness of a logistics network. Min et al. [28] proposed a mixed integer non-linear programming model to minimize the total reverse logistics costs for the reverse logistics problem involving both spatial and temporal consolidation of returned products. Fuente et al. [29] proposed an integrated model for supply chain management (IMSCM) in which the operation of the reverse chain had been built based on the existing processes of the forward chain.

Finally, Lee and Dong [30] develop an MILP model for integrated logistics network design for end-of-lease computer products. They consider a simple network with a single production center and a given number of hybrid distribution-collection facilities to be opened which they solve using tabu search. However, all of researches are found for some cost in reverse logistics. Our study focuses on a general framework and state total cost in reverse supply chain.

This paper propose a multi layers, multi product reverse supply chain problem which consist of returning center, disassembly center, processing center, manufacturing center, recycling center, material

center and distribution center and minimize the total costs in the reverse supply chain for returned products.

Problem definition and modelling

In forward logistics, suppliers offer raw materials to manufacturers. These manufacturers deliver finished products to distributors who finally distribute them to customers. In reverse logistics, collectors and recyclers play important roles for reuse, recycle, remanufacturing and disposal. The reverse supply chain under study is multi-layer, multi-product. In the designed (planned) model, the returned products after collecting and inspecting divides into two groups of disassembling and not disassembling products. The products which can be taken parted to the parts will be sent to the disassembling centers and there, they will convert to the parts. There they divide into reusable and not reusable parts. The not reusable parts will rebut safely and the reusable parts will be sent to the processing center. Some of the products that don't need to be disassembling; according to their variety will be transmitted to the processing center right after collecting centers, then considering to the variety of product and the request of manufacturing centers, will be sent to them. In the remanufacturing process, according to the production center's demand, the parts which can be used again, after processing center will be sent to the remanufacturing center and after compounding with the other parts will be changed into new products and can return to the distribution chain. In the recycling process according to the recycling center's demand the disassembled parts (which can recover again) right after disassembling centers will be sent to the recycling centers for the purpose of producing the secondary materials.

Purpose:

In this paper the reverse supply chain model has been considered for returned products with the purpose of minimizing the reverse supply chain costs.

Assumptions:
- The quantity of return, disassembly, processing, manufacturing, recycling, material and distribution centers are determined.
- Some products will transport straightly from return centers to the processing centers.
- Some parts will transport straightly from disassembly centers to the recycling centers.

Indices, Parameters, and Decision variables:

Indices:

i: index of returning centers

j: index of disassembly centers

k: index of processing center

f: index of manufacturing center

r :index of recycling center

w: index of material

p: index of products

m: index of parts

l: index of distribution centers

c: index of clients

Parameters:

a_{ip}: the capacity of returning center i for product p

b_{jm}: The capacity of disassembly center j for parts m

u_{km}: The capacity of processing center k for part m

d_{rm}: The capacity of recycling center r for part m

h_{fm}: The capacity of manufacturing center f for parts m

E_{lm}: The capacity of distribution center l for part m

DM_{fm}: the manufacturing center's demand f for part m

$DRCP_{rp}$: the recycling center's demand r for product p

$DRCM_{rm}$: the recycling center's demand r for part m

DD_{lm}: the distribution center's demand l for part m

DC_{cm}: the client's demand c for part m

DMA_{wm}: the material center's demand w for part m

n_{mp}: The produced part's amount m from disassembling one product p

$CSRD_{ijp}$: unit cost of transportation from returning center i to disassembly center j for product p

$CSRP_{ikp}$: unit cost of transportation from returning center i into the processing center k for product p

$CSDP_{jkm}$: unit cost of transportation from disassembly center j into processing center k for part m

$CSDRC_{jrm}$: unit cost of transportation from disassembly center j into the recycling center r for part m

$CSPM_{kfm}$: unit cost of transportation from processing center k into the manufacturing center f for part m

$CSPRC_{krm}$: unit cost of transportation from processing center k into the recycling center r for part m

$CSRCM_{rwm}$: unit cost of transportation from recycling center r into the material center w for part m

$CSPDC_{flm}$: unit cost of transportation from manufacturing center f into the distribution center l for part m

$CSDC_{lcm}$: unit cost of transportation from distribution center l into the clients c for part m

$FOCD_{jm}$: the fixed opening cost for disassembly center j for part m

$FOCP_{km}$: the fixed opening cost for processing centers k for part m

$FOCR_{ip}$: the fixed opening cost for returning centers i for product p

$FOCRC_{rm}$: the fixed opening cost for recycling centers r for part m

RMC_{fm}: unit cost of remanufacturing in manufacturing center f for part m

IC_{ip}: unit cost of maintaining in returning center i for product p

OCD_{jm}: unit cost of operations in disassembly center j for part m

OCP_{km}: unit cost of operations in processing center k part m

$OCRC_{rm}$: unit cost of operations in recycling center r part m

NRS_{min}: the minimum amount of returning center for opening and operations

NRS_{max}: the maximum amount of returning centers for operations and opening

NDS_{min}: the minimum amount of disassembling centers for opening and operations

NDS_{max}: the maximum quantity of disassembling centers for opening and operations

NPS_{min}: the minimum amount of processing centers for opening and operations

NPS_{max}: the maximum amount of processing centers for opening and operations

$NRCS_{min}$: the minimum amount of recycling centers for opening and operations

$NRCS_{max}$: the maximum amount of recycling centers for opening and operations

Decision variables:

φ_{ijp}: amount shipped from returning center i to disassembling center j for product p

δ_{ikp}: amount shipped from returning center i into the processing center k for product p

G_{jkm}: amount shipped from disassembly center j into the processing center k for part m

O_{jrm}: amount shipped from disassembly center j into the recycling center r for part m

Q_{kfm}: amount shipped from processing center k into the manufacturing center f for part m

S_{krm}: amount shipped from processing center k into the recycling center r for part m

ρ_{rwm}: amount shipped from recycling center r into the material center w for part m

T_{flm}: amount shipped from manufacturing center f into the distribution center l for part m

V_{lcm}: amount shipped from distribution center l into the clients c for part m

α_{jm}: if the disassembly center j is open for part m, 1 or otherwise 0

β_{km}: if processing center k is open for part m 1 or otherwise 0

γ_{ip}: if the returning center i is open for product p, 1 or otherwise 0

λ_{rm}: if recycling center r is open for part m, 1 or otherwise 0

μ_{fm}: the part's flow amount m in manufacturing center f

X_{ip}: the product's flow amount p in returning center i

Y_{jm}: the part's flow amount m in disassembly center j

θ_{km}: the part's flow amount m in processing center k

τ_{rm}: the part's flow amount m in recycling center r

Mathematical model

The formulation of the mathematical model is given below:

$$\text{Min} Z = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{p=1}^{P} csrd_{ijp} \Phi_{ijp}$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{p=1}^{P} csrp_{ikp} \delta_{ikp} + \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{M} csdp_{jkm} G_{jkm}$$

$$+ \sum_{j=1}^{J} \sum_{r=1}^{R} \sum_{m=1}^{M} csdrc_{jrm} O_{jrm} + \sum_{k=1}^{K} \sum_{f=1}^{F} \sum_{m=1}^{M} csrm_{kfm} Q_{kfm}$$

$$+ \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{m=1}^{M} csprc_{krm} S_{krm}$$

$$+ \sum_{r=1}^{R} \sum_{w=1}^{W} \sum_{m=1}^{M} csrcm_{rwm} \rho_{rwm}$$

$$+ \sum_{f=1}^{F} \sum_{l=1}^{L} \sum_{m=1}^{M} cspdc_{flm} T_{flm} + \sum_{l=1}^{L} \sum_{c=1}^{C} \sum_{m=1}^{M} csdc_{lcm} V_{lcm}$$

$$+ \sum_{j=1}^{J} \sum_{m=1}^{M} focd_{jm} \alpha_{jm} + \sum_{k=1}^{K} \sum_{m=1}^{M} focp_{km} \beta_{km}$$

$$+ \sum_{i=1}^{I} \sum_{p=1}^{P} focr_{ip} \gamma_{ip} + \sum_{r=1}^{R} \sum_{m=1}^{M} focrc_{rm} \lambda_{rm}$$

$$+ \sum_{f=1}^{F} \sum_{m=1}^{M} rmc_{fm} \mu_{fm}$$

$$+ \sum_{i=1}^{I} \sum_{p=1}^{P} ic_{ip} X_{ip} + \sum_{j=1}^{J} \sum_{m=1}^{M} ocd_{jm} Y_{jm}$$

$$+ \sum_{k=1}^{K} \sum_{m=1}^{M} ocp_{km} \theta_{km} + \sum_{r=1}^{R} \sum_{m=1}^{M} ocrc_{rm} \tau_{rm}, \tag{1}$$

s.t.

$$\sum_{j=1}^{J} \Phi_{ijp} \leq a_{ip} \gamma_{ip} \quad \forall i, p, \tag{2}$$

$$\sum_{k=1}^{K} \delta_{ikp} \leq a_{ip} \gamma_{ip} \quad \forall i, p, \tag{3}$$

$$X_{ip} \leq a_{ip} \gamma_{ip} \quad \forall i, p, \tag{4}$$

$$\sum_{k=1}^{K} G_{jkm} \leq b_{jm} \alpha_{jm} \quad \forall j, m, \tag{5}$$

$$\sum_{r=1}^{R} O_{jrm} \leq b_{jm} \alpha_{jm} \quad \forall j, m, \tag{6}$$

$$Y_{jm} \leq b_{jm} \alpha_{jm} \quad \forall j, m, \tag{7}$$

$$\sum_{f=1}^{F} Q_{kfm} \leq u_{km} \beta_{km} \quad \forall k, m, \tag{8}$$

$$\sum_{r=1}^{R} S_{krm} \leq u_{km} \beta_{km} \quad \forall k, m, \tag{9}$$

$$\theta_{km} \leq u_{km} \beta_{km} \quad \forall k, m, \tag{10}$$

$$\sum_{w=1}^{W} \rho_{rwm} \leq d_{rm} \lambda_{rm} \quad \forall r, m, \tag{11}$$

$$\sum_{l=1}^{L} T_{flm} \leq h_{fm} \quad \forall f, m, \tag{12}$$

$$\mu_{fm} \leq h_{fm} \quad \forall f, m, \tag{13}$$

$$\sum_{c=1}^{C} V_{lcm} \leq e_{lm} \quad \forall l, m, \tag{14}$$

$$\sum_{k=1}^{K} Q_{kfm} \geq DM_{fm} \quad \forall f, m, \tag{15}$$

$$\mu_{fm} \geq DM_{fm} \quad \forall f, m, \tag{16}$$

$$\sum_{f=1}^{F} T_{flm} \geq DD_{lm} \quad \forall l, m, \tag{17}$$

$$\sum_{l=1}^{L} V_{lcm} \geq DC_{cm} \quad \forall c, m, \tag{18}$$

$$\sum_{r=1}^{R} \rho_{rwm} \geq DMA_{wm} \quad \forall w, m, \tag{19}$$

$$\sum_{j=1}^{J} O_{jrm} + \sum_{k=1}^{K} S_{krm} \geq DRCM_{rm} \quad \forall r, m, \tag{20}$$

$$\tau_{rm} \geq DRCM_{rm} \quad \forall r, m, \tag{21}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} \delta_{ikp} \geq \sum_{r=1}^{R} DRCP_{rp} \quad \forall p, \tag{22}$$

$$\sum_{j=1}^{J} \sum_{k=1}^{K} G_{jkm} \geq \sum_{f=1}^{F} DM_{fm} \quad \forall m, \tag{23}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} G_{jkm} \leq n_{mp} \left(\sum_{i=1}^{I}\sum_{j=1}^{J} \Phi_{ijp}\right) \quad \forall m,p, \quad (24)$$

$$\sum_{j=1}^{J}\sum_{r=1}^{R} O_{jrm} \leq n_{mp} \left(\sum_{i=1}^{I}\sum_{j=1}^{J} \Phi_{ijp}\right) \quad \forall m,p, \quad (25)$$

$$NRS_{\min} \leq \sum_{i=1}^{I} \gamma_{ip} \leq NRS_{\max} \quad \forall p, \quad (26)$$

$$NDS_{\min} \leq \sum_{j=1}^{J} \alpha_{jm} \leq NDS_{\max} \quad \forall m, \quad (27)$$

$$NPS_{\min} \leq \sum_{k=1}^{K} \beta_{km} \leq NPS_{\max} \quad \forall m, \quad (28)$$

$$NRCS_{\min} \leq \sum_{r=1}^{R} \lambda_{rm} \leq NRCS_{\max} \quad \forall m, \quad (29)$$

$$\sum_{f=1}^{F} T_{flm} = \sum_{c=1}^{C} V_{lcm} \quad \forall l,m, \quad (30)$$

$$\sum_{k=1}^{K} G_{jkm} + \sum_{r=1}^{R} O_{jrm} \leq Y_{jm} \quad \forall j,m, \quad (31)$$

$$\sum_{j=1}^{J} \Phi_{ijp} + \sum_{k=1}^{K} \delta_{ikp} \leq X_{ip} \quad \forall i,p, \quad (32)$$

$$\sum_{f=1}^{F} Q_{kfm} + \sum_{r=1}^{R} S_{krm} \leq \theta_{km} \quad \forall k,m, \quad (33)$$

$$\sum_{w=1}^{W} \rho_{rwm} \leq \tau_{rm} \quad \forall r,m, \quad (34)$$

$$\sum_{l=1}^{L} T_{flm} \leq \mu_{fm} \quad \forall f,m, \quad (35)$$

$$\Phi_{ijp}, \delta_{ikp}, G_{jkm}, O_{jrm}, Q_{kfm}, S_{krm}, \rho_{rwm},$$
$$T_{flm}, V_{lcm}, \mu_{fm}, X_{ip}, Y_{jm}, \theta_{km}, \tau_{rm} \geq 0 \quad (36)$$
$$\forall i,j,k,f,r,w,p,m,l,c,$$

$$\alpha_{jm}, \beta_{km}, \gamma_{ip}, \lambda_{rm} = \{0,1\} \quad \forall i,j,k,p,m. \quad (37)$$

Objective function:

We want to demonstrate a model in reverse supply chain in a way to minimize the chain costs. We should introduce a model which minimizes the transportation cost of products and parts between centers and at the same time minimizes the fixed opening cost of sites and operation's cost on parts and supply maintenance costs and remanufacturing costs. By attention to the definition of Indices, parameters and

Decision variables; the objective function will be defined , which consists of : minimizing the costs of transportation of products and parts, the fixed opening cost of centers and operations costs on parts and the supply maintenance costs, remanufacturing costs in reverse supply chain (1).

Constraints:

(2,3) these constraints are decelerating that the amount of shipping products from any returning center (if it is opened) into the disassembly, processing centers for each product should be equal or smaller than the capacity of that returning center.

(4) This constraint is stating that the amount of products which will be collected in the returning center should be equal or smaller than the capacity of that returning center.

(5) and (6) these constraints are stating that the amount of sent parts from any disassembly centers and recycling centers should be equal or smaller than the capacity of the same disassembly center for each part.

(7) This constraint is stating that the amount of a part which is in the disassembly center should be equal or smaller than the capacity of the same disassembly center.

(8) and (9) these constraints are stating that the amount of shipping parts from any processing centers (if it is opened) into the manufacturing centers and recycling centers should be equal or smaller than the capacity of the same processing centers for each parts.

(10) this constraint is stating that the amount of a part which is in the processing center should be equal or smaller than the capacity of the same processing center.

(11) this constraint is stating that the amount of the parts which shipping from any recycling center (if it is opened) into the material centers should be equal or smaller than the capacity of the same recycling for each part.

(12) this constraint states that the amount of sent parts from any manufacturing center into the distribution centers should be equal or smaller than the capacity of the same manufacturing center for each part.

(13) this constraint states that the amount of part in each manufacturing center should be equal or smaller than the capacity of the same manufacturing center.

(14) this constraint states that the amount of sent parts from any distribution center to the client should be equal or smaller than the capacity of the same distribution center for clients.

(15) and (16) states the demand amount of manufacturing center for parts.

(17) states the part demand amount of distribution centers.

(18) indicates the client's part demand amount.

(19) states the part demand amount of material center.

(20) and (21) states the part demand amount of recycling centers.

(22) and (23) states that the manufacturing and recycling center's demand, is for products and parts which are transported from the returning and disassembly centers into the processing center.

(24) and (25) these constraints are related to the balance of parts flow from the disassembly of products.

(26)–(29) these constraints are stating that the min and max index amount of returning , disassembling, processing and recycling centers.

(30) this constraint states that the amount of sent parts from manufacturing centers to the distribution center is equal to the sent parts from distribution centers in to the client.

(31) this constraint states that the amount of sent parts from each disassembly center into the processing and recycling centers should be equal or smaller than the parts amount in that disassembly center.

(32) these constraint states that the amount of sent products from each returning center into the disassembly, processing centers, should be equal or smaller than the product's amount in that returning center.

(33) these constraint state that the amount of sent parts from each of the processing centers into the manufacturing and recycling centers should be equal or smaller than the flow amount of parts in that processing center.

(34) this constraint states that the sent parts amount from any recycling center into the material centers should be equal or smaller than the parts amount in that recycling center.

(35) this constraint states that the sent parts amount from any manufacturing center into the distribution centers should be equal or smaller than the parts flow amount in that manufacturing center.

(36) and (37) enforce the binary and non- negativity restrictions on the corresponding decision variables.

Numerical experiment

We solved the presented mathematic model by using Lingo 9, which is an operation research software in this multi layers and multi products model,

Table 1

Numerical results using LINGO 9 Software.

$\varphi(2,1,4)$	1.2	$\rho(3,1,2)$	3	$\beta(4,3)$	1
$\varphi(2,3,2)$	0.9	$\rho(3,2,2)$	8	$\theta(1,1)$	71
$\varphi(3,3,1)$	0.9	$\rho(3,3,1)$	18	$\theta(1,2)$	57
$\varphi(3,3,3)$	1.1	$\rho(3,3,2)$	14	$\theta(2,2)$	16
$\delta(2,1,2)$	34	$T(1,1,3)$	6	$\theta(2,3)$	63
$\delta(2,4,4)$	28	$T(1,2,1)$	8	$\theta(3,1)$	8
$\delta(3,1,1)$	41	$T(1,2,3)$	13	$\theta(4,2)$	20
$\delta(3,4,3)$	8	$T(1,3,2)$	3	$\theta(4,3)$	7
$G(1,3,2)$	62	$T(2,1,2)$	7	$\gamma(2,1)$	1
$G(1,4,3)$	50	$T(2,2,3)$	17	$\gamma(2,2)$	1
$G(2,1,3)$	16	$T(2,3,1)$	24	$\gamma(2,3)$	1
$G(3,2,1)$	59	$T(2,3,3)$	16	$\gamma(2,4)$	1
$O(1,1,3)$	8	$T(3,2,1)$	8	$\gamma(3,1)$	1
$O(1,3,2)$	18	$T(3,2,2)$	8	$\gamma(3,2)$	1
$O(1,3,3)$	4	$T(4,1,1)$	13	$\gamma(3,3)$	1
$O(2,2,3)$	4	$T(4,2,1)$	3	$\gamma(3,4)$	1
$O(2,3,1)$	18	$T(4,2,2)$	5	$X(2,2)$	34.87
$Q(1,1,1)$	9	$T(5,1,1)$	8	$X(2,4)$	29.19
$Q(1,1,2)$	17	$T(5,1,2)$	16	$X(3,1)$	41.94
$Q(1,2,1)$	18	$T(5,3,3)$	17	$X(3,3)$	9.09
$Q(1,2,2)$	16	$V(1,1,2)$	6	$\lambda(1,1)$	1
$Q(1,3,2)$	8	$V(1,1,3)$	6	$\lambda(1,2)$	1
$Q(1,4,1)$	16	$V(1,2,2)$	17	$\lambda(1,3)$	1
$Q(1,4,2)$	5	$V(1,3,1)$	1	$\lambda(2,1)$	1
$Q(1,5,1)$	8	$V(1,4,1)$	20	$\lambda(2,2)$	1
$Q(2,1,3)$	19	$V(2,1,2)$	4	$\lambda(2,3)$	1
$Q(2,3,3)$	14	$V(2,2,3)$	16	$\lambda(3,1)$	1
$Q(2,4,3)$	9	$V(2,3,1)$	19	$\lambda(3,2)$	1
$Q(2,5,2)$	16	$V(2,4,2)$	9	$\tau(1,1)$	32
$Q(2,5,3)$	17	$V(2,4,3)$	14	$\tau(1,2)$	20
$Q(3,3,1)$	8	$V(3,1,1)$	20	$\tau(1,3)$	12
$Q(4,2,3)$	7	$V(3,1,3)$	13	$\tau(2,1)$	17
$S(1,1,1)$	16	$V(3,2,1)$	4	$\tau(2,2)$	11
$S(1,2,1)$	4	$V(3,3,2)$	3	$\tau(2,3)$	15
$S(1,2,2)$	11	$V(3,3,3)$	20	$\tau(3,1)$	18
$S(2,1,3)$	4	$\alpha(1,2)$	1	$\tau(3,2)$	25
$S(4,1,2)$	20	$\alpha(1,3)$	1	$\tau(3,3)$	4
$\rho(1,1,1)$	1	$\alpha(2,1)$	1	$\mu(1,1)$	9
$\rho(1,1,2)$	12	$\alpha(2,2)$	1	$\mu(1,2)$	17
$\rho(1,2,1)$	17	$\alpha(2,3)$	1	$\mu(1,3)$	19
$\rho(1,2,3)$	6	$\alpha(3,1)$	1	$\mu(2,1)$	24
$\rho(1,4,1)$	14	$Y(1,2)$	80	$\mu(2,2)$	16
$\rho(1,4,3)$	2	$Y(1,3)$	62	$\mu(2,3)$	33
$\rho(1,5,2)$	8	$Y(2,1)$	18	$\mu(3,1)$	8
$\rho(1,5,3)$	4	$Y(2,3)$	20	$\mu(3,2)$	8
$\rho(2,1,3)$	1	$Y(3,1)$	59	$\mu(3,3)$	14
$\rho(2,3,1)$	1	$\beta(1,1)$	1	$\mu(4,1)$	16
$\rho(2,3,3)$	1	$\beta(1,2)$	1	$\mu(4,2)$	5
$\rho(2,4,2)$	4	$\beta(2,2)$	1	$\mu(4,3)$	9
$\rho(2,5,1)$	16	$\beta(2,3)$	1	$\mu(5,1)$	8
$\rho(2,5,2)$	7	$\beta(3,1)$	1	$\mu(5,2)$	16
$\rho(2,5,3)$	13	$\beta(4,2)$	1	$\mu(5,3)$	17

we are attempting to minimize the costs of fixed opening facilities, transportation and shipping of products and parts between centers and also the operations, supply maintenance and remanufacturing costs, and also the product amount and sending parts into the centers and the amount of it would be calculated. To analyzing the suggested model we create numerical example in small size and then solve the created example by lingo software.

In small size we consider the index quantities as variables between 3 to 5 to solve the problem, so we replace the inputs of problem in the model and by using the lingo we will solve the problem and finally; the model solving outputs and the objective function amount and the implementation time of it would be demonstrate. By attention to the inputs of the model and solving it, the outputs of model and objective function amount and the implementation time has been obtained which are as follow; The obtained objective function is 29653.20 which obtained in zero time all the variables which were not zero 0 quantities are shown in Table 1. After solving the model we will find out that the decision variable $\alpha(1,2)$ gained 1 quantity. This means that the disassembly center 1 should be opened for part 2. The decision variable $\lambda(3,2)$ obtained 1, means that the recycling center 3 would be opened for part 2. Generally when the decision variables α_{jm}, β_{km}, γ_{ip}, λ_{rm} gained 1, it indicates that the considered center to that decision variable will be opened for that part or product. The decision variable $Q(1,4,2)$ is considered 5. This means that the amount of part 2 from processing center 1 into the manufacturing center 4 is 5. The decision variable $\tau(2,3)$ got 15, it means that the amount of part 3 in recycling center 2 is 15. $\rho(3,2,2) = 8$ means that the amount of part 2 from recycling center 3 into the material center 2 is 8.

Sensitivity analysis

According to the results that obtained from the mathematical model in different dimension we want to evaluate the importance of decision variables. This importance of decision variables is determined by new elimination method. In this method after elimination of each decision variables and rerunning the model, the importance of the variable would be clear. It is not possible to remove all decision variables in

the model because some of the variables in the model are important so that the elimination of the variables will cause the closure of facilities in the model. Changes of decision variables after removal variables δ_{ikp}, G_{jkm}, O_{jrm}, S_{krm}, ρ_{rwm} is shown in Table 2.

For instance after removal of decision variable G_{jkm} that represent the amount shipped from disassembly center j into the processing center k for part m and rerunning the small size of the model we find the value of the objective function is changed to 26622.4 and the value of the decision variables φ_{ijp}, T_{flm}, Y_{jm}, X_{ip}, μ_{fm} are changed. Figure 1 shows the changes in the decision variable φ_{ijp}.
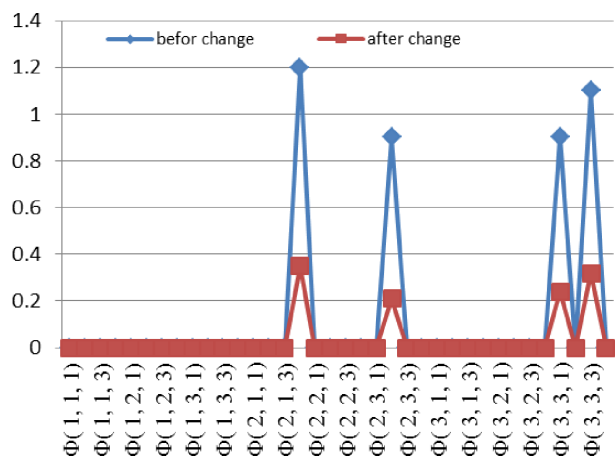

Fig. 1. Graphical representations of changes in the decision variable φ_{ijp}.

According to the Sensitivity Analysis we perceive that the decision variables G_{jkm}, O_{jrm}, S_{krm} are important than the decision variables δ_{ikp}, ρ_{rwm}. Just decision variable X_{ip} will be changed after removal the decision variable δ_{ik} and decision variables λ_{rm}, τ_{rm} will be changed after removal the decision variable ρ_{rwm} whereas five decision variables φ_{ijp}, O_{jrm}, S_{krm}, α_{jm}, Y_{jm} will be changed after removal the decision variable G_{jkm}. Seven decision variables G_{jkm}, Q_{kfm}, S_{krm}, T_{flm}, β_{km}, Y_{jm}, θ_{km} will be changed after removal the decision variable O_{jrm}. Eight decision variables G_{jkm}, O_{jrm}, T_{flm}, α_{jm}, β_{km}, μ_{fm}, Y_{jm}, θ_{km} will be changed after removal the decision variable S_{krm}. The result of performed analysis has many applications in strategic decision making.

Table 2
Changes of decision variables after removal variables.

Decision variables	δ_{ikp}		G_{jkm}		O_{jrm}		S_{krm}		ρ_{rwm}	
φ_{ijp}			✓	$\varphi(2,1,4)$ 0.35 $\varphi(2,3,2)$ 0.21 $\varphi(3,3,1)$ 0.24 $\varphi(3,3,3)$ 0.32						
G_{jkm}					✓	G(1,4,3) 62 G(2,1,3) 4	✓	G(1,4,3) 46 G(2,1,3) 20		
O_{jrm}			✓	O(1,1,3) 12 O(2,2,3) 0			✓	O(1,1,3) 12 O(2,2,1) 4 O(2,2,2) 11 O(3,1,1) 16 O(3,1,2) 20		
Q_{kfm}					✓	Q(1,2,1) 17 Q(1,5,3) 8 Q(2,2,1) 1 Q(2,5,3) 9				
S_{krm}		1,2,1	✓	S(2,1,3) 0 S(4,2,3) 4	✓	S(1,1,1) 0 S(1,3,1) 18 S(1,3,3) 4 S(2,1,1) 16 S(2,1,3) 12 S(2,3,2) 18 S(4,2,3) 4				
T_{flm}					✓	T(1,3,1) 1 T(2,3,1) 23	✓	T(1,3,1) 1 T(2,3,1) 23		
α_{jm}			✓	$\alpha(2,3)$ 0 $\alpha(3,2)$ 0 $\alpha(3,3)$ 1			✓	$\alpha(2,3)$ 0 $\alpha(3,2)$ 0		
β_{km}					✓	$\beta(1,3)$ 1 $\beta(2,1)$ 1	✓	$\beta(4,2)$ $\beta(4,3)$ 1		
λ_{rm}									✓	$\lambda(1,1)$ 0 $\lambda(1,2)$ 0 $\lambda(2,2)$ 0 $\lambda(2,3)$ 0 $\lambda(3,1)$ 0
μ_{fm}							✓	$\mu(2,1)$ 23		
X_{ip}	✓	X(2,2) 0.9 X(2,4) 1.2 X(3,1) 0.9 X(3,3) 1.1								
Y_{jm}			✓	Y(1,2) 18 Y(1,3) 16 Y(2,1) 18 Y(2,3) 0 Y(3,1) 0	✓	Y(1,2) 62 Y(2,1) 0 Y(2,3) 4	✓	Y(2,1) 22 Y(2,2) 11 Y(2,3) 24 Y(3,1) 75 Y(3,2) 20		
θ_{km}					✓	$\theta(1,1)$ 72 $\theta(1,3)$ 12 $\theta(2,1)$ 17 $\theta(2,2)$ 34 $\theta(4,3)$ 11	✓	$\theta(1,1)$ 51 $\theta(1,2)$ 46 $\theta(2,3)$ 59 $\theta(4,2)$ 0		
τ_{rm}									✓	$\tau(1,1)$ 16 $\tau(2,1)$ 4 $\tau(2,3)$ 4 $\tau(3,2)$ 18

Conclusions

In this paper, a reverse supply chain was considered minimizing the total cost of transport, inspection, remanufacture and maintenance. The presented model was an integer linear programming model for multi-layer, multi-product reverse supply chain that minimized the products and parts transportation costs among centers and also sites launch, operation parts, maintenance and remanufacturing costs at the same time. We solved the proposed model using Lingo 9 software. A comprehensive sensitivity analysis was studied to validate the proposed mathematical model and the obtained results.

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