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FILLING MODEL FOR THE WORKING MULTI-DISC BIOMASS GRAIN GRINDING UNIT

Efficiency, functionality and performance of the grain grinding process are significantly influenced by phenomena that are difficult to describe and occur in the working area of the grinder. In a machine-based, multi-disc grinding of grain biomaterials, the design of the quasi-cutting unit, volumes, sections of transport/grinding holes, their motion and the design features of the discs (the grinding unit) must facilitate the functions of grinding in the inter-hole space (with minimum energy-consumption of the process and maximum efficiency) and minimising undesirable phenomena related to mixing and transport.

The pre-requisite for optimisation of the quasi-cutting unit design is a mathematical model. Among many aspects of the problem, this study describes a sample procedure resulting in a filling model for a biomass grain quasi-cutting unit including an initial verification of the same under conditions of the evaluation of maize and triticale grain grinding efficiency, using an innovative multi-hole 5-disc and 7-disc grinder.

1. Introduction

The grinding/crushing machines used in industrial practice to crush solid biomaterials are characterised by small technical efficiency (usually a few to several per cent) in the realised process and, what is also connected to this, a large consumption of energy – which is particularly important if there are large quantities of biomaterial to be grinded (e.g. in the power engineering industry, grain mills).

The crushing method and the machinery used for this purpose should be adapted to the type of the material being grinded and, in particular, to its mechanical properties [2, 5, 9]. The main differentiation can be observed

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between the organic materials (of plant and animal origin) and mineral matter (aggregates, minerals).

In the most general understanding, the grinding of any solid biomaterial is the breaking up of its mass into parts. The breaking up of a material into parts of required dimensions is made after overcoming the internal material cohesion forces. So, we can define grinding as the discretisation of the material, because the dimensions of broken-up pieces (of the product or fed material) differ "in steps" – discontinuously. Grinding usually does not refer to the breaking up of a big uniform mass of material, but a material which is already broken-up into parts (in pieces), either because of (natural or forced) coarse crushing – e.g. mineral aggregates, mines, or because the material occurs broken-up (in pieces) in its natural environment – e.g. organic (plant) grainy biomaterials. The measure (factor) of the breaking up (disintegration) of the material into parts is the so-called degree of fineness (degree of discretisation) [5, 9], which is generally defined as the quotient of dimensions of quantities characterising a material piece before (A) and after quasi-cutting (a), which is defined by the general relationship:

$$\lambda = \frac{A}{a}. \tag{1}$$

The quantities which characterise the particles of broken-up material may be: mass, volume, surface or linear dimensions (e.g. diameter, length, width), either actual ones or the so-called substitute ones. As a result, there may be quantities referring to the discretisation of a material in the form of the degree of fineness: mass fineness, surface fineness or linear fineness [2, 5, 9].

The effectiveness of biomaterial quasi-cutting depends mainly on the generally-understood susceptibility of the material to quasi-cutting, and it (the effectiveness) can be measured through the amount of consumed energy or the work necessary to realise the cutting process [5]. The susceptibility of a biomaterial to quasi-cutting is, in particular, connected with a loss of its cohesion.

The amount of energy used for quasi-cutting of a given biomaterial depends on:

- Properties of the biomaterial
- Degree of fineness
- Method of cutting, grinding or crushing
- Conditions of cutting, grinding or crushing.

Each of the factors affecting the amount of energy used for biomass quasi-cutting is characterised by the following quantities and properties:

a) Properties of the material
Strength of the material (values of boundary stresses and forces) referring to:
– cohesion (non-cohesion, destruction, disintegration),
– plasticity (plastic strains),
– elasticity (elastic/reversible strains),
b) The value of boundary stresses is connected with the types of properties concerning the strength of the biomaterial being quasi-cut up, e.g.:
– tearing, squeezing,
– shearing,
– surface pressures,
– impact resistance,
– bending, breaking,
– grindability (from external to internal friction).
c) Degree of fineness
– The sizes of biomaterial particles before and after quasi-cutting (mass, volume, surface, linear dimension).
– The value of energy of the free surface of biomaterial.
– Resistance of the biomaterial to non-cohesion.
d) Method (process) of cutting
– Taking into account and making use of the phenomena accompanying and arising during quasi-cutting (e.g. breaking, abrasion, crushing, cutting, striking), and referring, in particular, to energy dissipation (e.g. heat, friction).
– Structural solution for a grinding machine making use of the physical phenomena involved in cutting (type of machines and devices).
e) Conditions of cutting
– The interaction and influence of the environment (natural and technical conditions); temperature, moisture (primary conditions).
– The effects of the cutting process; temperature, moisture, friction, acoustic waves, vibrations (secondary conditions).

The cutting, grinding or crushing process is described by numerous hypotheses [4, 5] concerning the relations between the energy (work) needed for quasi-cutting and the relevant characteristic quantities, in particular, referring to the degree of fineness (usually described by means of geometrical quantities) and biomaterial properties. The hypotheses (theories) applied generally do not make allowances for the methods and conditions of quasi-cutting.

The subject of this study is an innovative concept of quasi-cutting proposed in the multi-edged grinder. The concept increases the effectiveness of the quasi-cutting process through its construction, making special use of the cutting phenomenon occurring during processing – it determines the occurrence of the quasi-cutting.

The primary objective of the modelling process was to obtain a mathematical relationship that would enable answering the problem question related to the quasi-cutting unit filling: what factors, design/operational features and how they affect:

- The dynamics of the movement of the biomass grains through the working unit of the grinder,
- The efficiency of the grain biomass grinding,

with a multi-disc grinder constructed according to a patent of the University of Technology and Life Sciences in Bydgoszcz used as an example [4]. Also, the purpose of the study is to describe the complexity of phenomena, processes and relations of the multi-disc grinding.

2. Principles

For the purpose of this study, an assumption was made that the output model of the feed grain biomaterial is the substance disintegrated in one dimension and that grains are always positioned along its long axis, perpendicular to the cutting direction.

The state of the substance is described by the distribution of grain length probability. Because the material in the holes of the same disc is subject to the same cutting process in each hole, the state is indexed with the disc number (n) and cut number (m):

$$\rho_n^m : (0, l_{\max}] \to [0,1]\,, \qquad \int_0^{l_{\max}} \rho_n^m dl = 1. \tag{2}$$

Therefore, the movement to the next disc involves the quasi-cutting process, $n \leq m$. The state of the material in the first disc, before the first cut, corresponds to ρ_0^0 and is the initial state of the material. It can therefore be described with a certain function involving l_{max} value.

The state of the material changes as a result of the two mechanisms – grinding and removing grains of the target or smaller than target size from the device. An assumption was made at that stage that the angle of repose of the material does not depend on the state of the material.

Ultimately, the substance model should take into account three degrees of freedom of the grain orientation. In this case, the volume occupied by the same mass of material will vary depending on the distribution of grain size and shape. Also, the angle of repose may depend on the state of the material.

3. Modelling of the working grinding unit filling

There are two groups of factors directly affecting the mechanical processing and grinding of materials: the physio-chemical properties of the ground material, and the structural and operating parameters of the grinder [3,6].

To verify the proposed theoretical solutions, test were carried out using the following grinders: a 5-disc multi-hole RWT-5KZ grinder (Fig.1A and Fig.1B) , and a 7-disc RQS-7T grinder (Fig.1C), both owned by the research laboratory of the Technical Systems and Environment Protection Department, Institute of Production Techniques, the University of Technology and Life Sciences in Bydgoszcz (their basic parameters are presented in Table 1). The RWT-5KZ grinder working unit comprises five multi-hole grinding discs (Fig.1). Each disc is coupled individually with the drive motor through belt and gear transmissions. The grinder drive unit consists of five 3-phase electric motors controlled and powered by the pDrive inverter system.

Table 1.

The technical parameters of grinders: the RWT-5KZ 5-disc grinder and the RQS-7T 7-disc grinder

Structural and technical feature	RWT-5KZ	RQS-7T
Number of discs	5	7
Number of holes in disc	62-253	185
Number of hole rows in disc	1-3	1-3
Clearance between discs, mm	0.02-0.18	0.2-0.8
Hole diameters, mm	15-26	15.0; 17.5; 20.0
Hole arrangement radius, mm	85-114.5	65-92
Angular speed, rad/s	0-8.35	0; 28.5; 36.5
Disc thickness, mm	20	3

The grinding operation in multi-disc grinders is performed on the edges of the grinding holes as a result of the differences between angular speeds of the adjacent discs (Fig.3). Formation and separation of grain particles in multi-disc grinding is accomplished not only by cutting off of part of the feed material. The experiments to date performed by the author have shown that this is a complex process and the ground product is cut off from the original piece only in some cases. Most particles are formed as a result of the phenomena combining, among other things, cutting, breaking and tearing [13]. Therefore, due to the many aspects related to the issue of disintegration, the working unit of the biomass grain multi-disc grinder was called the quasi-cutting unit.

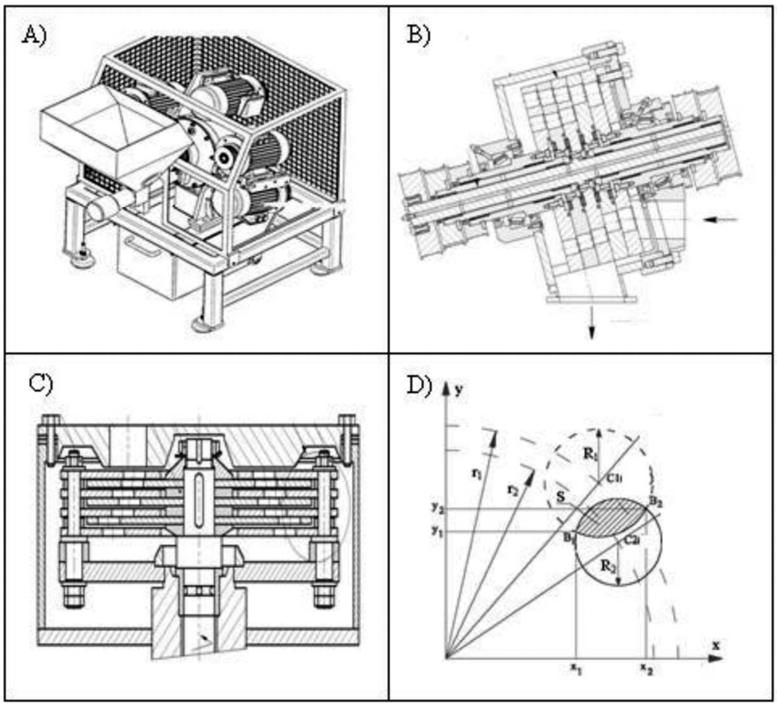

Fig. 1. RWT-5KZ multi-disc grinder; A) general view; B) RWT-5KZ 5-disc grinder working unit; C) RWT-5T 7-disc grinder working unit; D) Common area of two holes, grinding cross-section between the edges of the adjacent grinding holes: C_1, C_2 – working holes centres, R_1, R_2 – working holes radii, r_1, r_2 – working holes positioning radii, x_1, x_2, y_1, y_2 – co-ordinates for B_1 and B_2 quasi-cutting pair circles intersection points, S_C –quasi-cutting hole pair common area

One of the conditions required for even and effective grinder operations is a constant and steady feeding of material into the grinder working space. In the case analysed, this is performed by the screw feeder fitted directly under the charging hopper.

The grinding material comprised triticale and maize grains with stabilised parameters (Table 2). Biomass grains are heterogeneous material [3,10,12]. They consist of: hull (14%), embryo (2%) and endosperm (84%). Each of the components is characterised by different structural and mechanical properties. For example, while grinding the endosperm, which is characterised by high brittleness, it is cutting that occurs mostly. The grain hull is pliable and is torn apart. The hull fracture stress, depending on the type of grain and its moistness, is 0.96-30.8 MPa, whereas the endosperm fracture stress is 1.8-3.3 MPa, i.e. ca. 10 times lower than the hull fracture stress. It can be, therefore, concluded that the more compact the material, the more energy is needed to grind it [7,11,12].

Table 2.

Parameters of grains

Parameter	Triticale	Maize
Moisture content [%]	12.31	13.75
Angle of repose [°]	30	28
Friction angle (against steel surface)	21	20
External friction co-efficient (movement), grain/steel	0.375	0.321
Internal friction co-efficient	0.511	0.641
Resistance to compression [MPa]	13.7	11.3
Resistance to cutting [MPa]	68	55
Weight of 1000 grains [g]	32.50	366.10
Specific gravity [kg/m^3]	1355	1211
Porosity [%]	40	45

The examined biomass grains were measured in terms of size and geometric shape. The results are presented in Table 3. The resulting data indicate that the triticale grains are particles which meet shape specific requirements for oval grains to be used in the feedstuff industry (length/3 ≤ width ≈ thickness), whereas results for maize grains confirm their round shape. Irregular shapes dominate in both cases.

Table 3.

Size and geometric shape grains

Parameter			Triticale grains	Maize grains
Dimension	Length	Max. [mm]	8.10	11.90
		Min. [mm]	6.10	8.85
		Average. [mm]	7.203	10.660
		Standard deviation	0.543227	0.696348
	Width	Max. [mm]	3.75	10.35
		Min. [mm]	2.15	6.50
		Average. [mm]	2.929	8.978
		Standard deviation	0.332654	0.676172
	Thickness	Max. [mm]	3.40	8.35
		Min. [mm]	2.10	4.70
		Average. [mm]	2.663	5.745
		Standard deviation	0.247068	0.699825

3.1. Conditions of biomass grain movement

The feed grain biomaterial movement in the multi-disc space is uneven due to the nature of the design and the feed material. It begins when the area of the working holes positioned on the surface of the two adjoining discs overlap and the area of their common part (S_c) (Fig. 1D and Fig.4) starts to increase. Feed material movement from the preceding hole to the next hole (in the next disc) results from the effect of the longitudinal force caused by the screw line of the screw feeder and the effective component of the gravitational force. The cutting operation starts as soon as the feed material completely fills the decreasing common section of the adjacent holes. This analysis is based on the following simplifications:

1. The hole cross-section is a convex set (Fig.2).
2. The hole's volume is a cylinder (Cartesian product of the hole's cross-section and the disc thickness), (Fig.3).
3. Working hole on (n) disc can only connect to one hole in the adjacent disc (n+*1*) at any one time with passage to the disc (n+*2*), closed (Fig.4).

Moreover, the following parameters and designations of the same have been used in the mathematical analyses:

- thickness of n disc, y_n,
- minimum height (at the disc end), up to which the material fills the hole in n disc after k cut $\tilde{y}_n^{(k)}$,
- volume of the grain biomaterial in the hole of n disc after m cut, v_n^m.

For verification of the conditions, and their practical fulfilment, an analysis was made for a device fitted with a working unit with all discs identified with even numbers having the same angular speed of ω_0, while discs identified with odd numbers also having the same angular speed of ω_1.

The analysed speed values may have: the same signs (adjacent rotating discs rotating in the same direction), the opposite signs (adjacent discs rotating in the opposite direction) or alternating discs that are not rotating (having an angular speed equal to 0), and the working movement is carried out by their adjacent discs. The analysis concerns a ring at the grinding disc (Fig.2), with an internal radius of $R - r$, and an external radius of $R + r$. Within the analysed field, there are three equally distanced holes with r radius. The relation between these radii:

$$r = R \cdot \sin \frac{360}{2 \cdot 12}$$

that ensures that 12 such holes tangent to each other fit within the ring. Disc No. 2 is the same as disc No. 0 but it is rotated by 60°. Disc No. 1 is rotated in relation to disc No. 0 by 30 degrees, while disc No. 3 – by 90 degrees. The analysis shows that at any given moment, a particular hole of the first

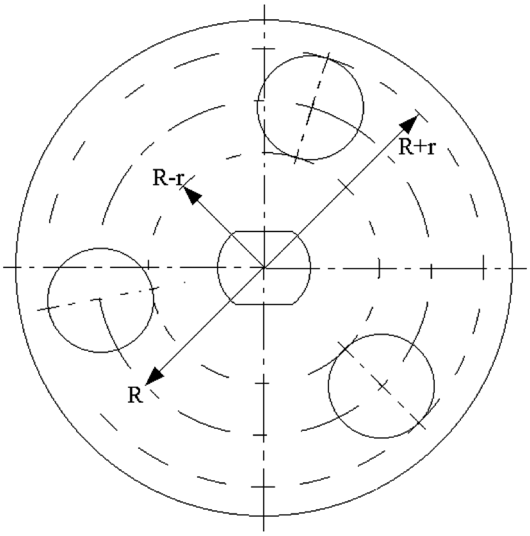

Fig. 2. Arranging the grinding holes in the rotating discs

disc is open for only one hole in disc No. 0 or No. 2. The situation is similar as regards disc No. 3.

The problem with such an arrangement is due to the fact that the cutting process will occur in three holes at the same time and the power consumed by such a system will be very changeable during the period. Therefore, to reduce dynamic irregularity of the grinder operation, the arrangement of the working holes in the discs must be designed in a way that ensures their more even distribution, e.g. by adding the virtual rings, described herein, with holes arranged in a similar way, if possible, with a different number of holes.

Filling of the hole in specific discs is presented in Fig. 4. As shown in the diagram, k cut after $(n-1)$ disc occurs earlier than k cut after n disc. With such a numbering of the cuts (numbering on each disc starts with the first cut), the grain may be subject to the cut of the same number at each disc boundary.

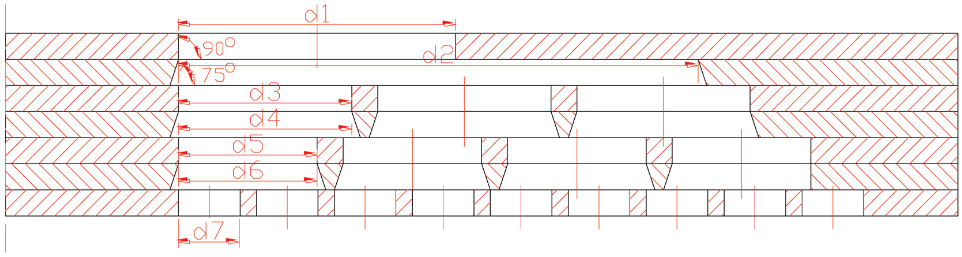

Fig. 3. Diagram of movement of the ground material in individual layers, arrow indicates cutting

3.2. Conditions of unit filling and efficiency

Since the main axis of the working unit of the grinder (device) is not vertical (Fig. 1), the direction of gravitational force is not perpendicular to the plane of the grinder rotating discs. The angular speed of the discs results in an additional component of the centrifugal acceleration. The acceleration sum within the hole is an effective gravitational acceleration. Its direction depends on the phase of the disc hole's rotation.

At that stage of the analysis, a zero angle of repose was used for the ground medium for simplification purposes. For such a simplification, the surface of the material in the working hole at any given moment of its movement is perpendicular to the direction of the vector of the effective gravitation within the hole's area. An assumption was also made that a variability of effective gravitation within the hole's area will not be taken into consideration and the surface of the material will be treated as flat within the hole's area[1].

When the common part of areas of the top and bottom holes reaches its maximum value, the common part starts to diminish. At this moment, in general cases, the cutting does not occur yet, because the whole area of the common part does not yet have to be filled with the material. Therefore, such a phase of relative motion must take place when the whole common part of the hole's cross-sections is filled with the material. The following simplifications are applied (Fig. 4):

1. A specific point of its cross-section is identified for each hole. It may be, for example, its geometric centre. The phase of relative motion of two holes may be described by specifying the angular distance α between the hole's centres. Range ends α are identified as α_p, α_k. Functions D, G, and CW have been defined that give for specific $\Delta\alpha$, an area of respectively the next (bottom) hole, the previous (top) hole, and their common part. Symbol α_{max} is used to identify the value of the angular distance, for which function $|CW|$ takes the maximum value (if the maximum is reached in more than one value of α, the highest of them is used as α_{max}). The average vector of effective gravitation is identified with $\vec{g}_D$, $\vec{g}_G$ (respectively for bottom and top hole). For simplification purpose, an assumption has been made that centrifugal components $\vec{g}_D$, $\vec{g}_G$ are mutually parallel (as a result of the difference in angular speed of two adjacent discs they differ in value only). V^G, V^D, S^G, S^D will be used to identify the volume of the previous and the next hole and the cross-sectional area of the previous and the next hole.

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[1] Within the hole's area, the surface will not be ideally flat; it will be the area of a paraboloid of revolution. Its curvature diminishes as we get closer to the revolution axis

2. For each vector $\vec{g} \in \mathbb{R}^3$, the plane $H_{\vec{g}}$ has been specified perpendicular to the vector, tangent to the boundary of a set $CW(\alpha)\times\{\ 0\}$. There are two such planes – one passes below set $CW(\alpha)\times\{\ 0\}$, and the other – above it.[2] The plane that passes above the vector was selected. It represents the surface of the material when the cutting begins
3. The cylinder $G(\alpha)\times[0, y_n] \subset \mathbb{R}^3$ has been divided with the plane $H_{\vec{g}G}$. The volume obtained under the plane has been identified as

$$V^G_{\vec{g}G}(\alpha) = \int_G \min\left\{y_n, H_{\vec{g}G}(r)\right\} d^2 r \tag{3}$$

   This function attributes to the phase of relative motion, the volume that remains in the previous hole when cutting begins.
4. Similarly, the plane $H_{\vec{g}}$ has been used to divide the cylinder $D(\Delta\alpha)\times[-y_{n+1}, 0] \subset \mathbb{R}^3$. The volume obtained below the plane has been identified as

$$V^D_{\vec{g}D}(\alpha) = \int_D \min\left\{y_{n+1}, H_{\vec{g}D}(r) + y_{n+1}\right\} d^2 \tag{4}$$

   This function attributes to the phase of relative motion, the volume that remains in the next hole when the cutting begins.
5. Expression $\left(V^G_{\vec{g}G}, V^G_{\vec{g}GD}\right)(\alpha)$, for range $\alpha \in (\alpha_{max}, \alpha_k)$ is a decreasing function. By reversing the function on its image, we obtain the phase of motion at which cutting begins for a specified volume of the material in both grinding holes.

$$\alpha_c(V) = \left(V^G_{\vec{g}G} + V^D_{\vec{g}D}\right)^{-1}(V) \tag{5}$$

   The function has been extended onto the set $[0, V^G + V^D]$, placing α_k to the left and α_{max} to the right of the original domain of the function.
6. The surface area of the common part when cutting begins for a specified volume:

$$S_c = \left|\left(Cw \cdot \left(V^G_{\vec{g}G} + V^D_{\vec{g}D}\right)^{-1}\right)\right| : \left|O, V^G + V^D\right| \to \mathbb{R} \tag{6}$$

   This is an increasing function – as the quantity of the feed material in both working holes increase, so is the rate at which cutting begins, so with the greater surface area of the common part of cross-sections.

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[2] The existence of exactly two tangent planes results from the convexity of the common part of cross-sections

The function obtained may be treated as a function of four variables – volume of the material in both holes, direction of effective radial gravitation component and the value of effective radial gravitation components in both holes S_c: $[0, V^G + V^D] \times S^1 \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$. The function must be determined numerically, and treated as coding of the influence of the hole's shape on the quasi-cutting process at various directions of effective gravitation vectors.

3.3. Changes in unit filling during quasi-cutting

A further analysis was carried out, taking account of the ground medium's angle of repose. It was assumed, for simplification purposes, that the angle of repose is constant in the function of the working displacement of ground material within the multi-disc area. Moreover, it was assumed that the material is moved only in the movement direction (movement direction of the preceding hole in relation to the next hole). This means that the material plane bends towards the bottom at an angle of repose in the direction in which the material is pushed. Since the direction of relative motion of the holes is always perpendicular to the direction of centrifugal acceleration, the vector of effective gravitation is modified by being rotated at the angle of repose with respect to the centrifugal component.

In a cylindrical co-ordinate system with base vectors $\{e_r, e_\varphi, e_z\}$, the effective gravitation vector has the following coordinates $(\omega^2 r,\ 0,\ g)$. After being rotated with respect to the radial component at γangle, the effective gravitation vector has the following components $(\omega^2 r,\ \pm\ g\sin\gamma, g\cos\gamma)$. Sign " – " indicates a situation when the previous disc rotates with respect to the next disc with a positive angular speed, and " + " sign – the opposite situation.

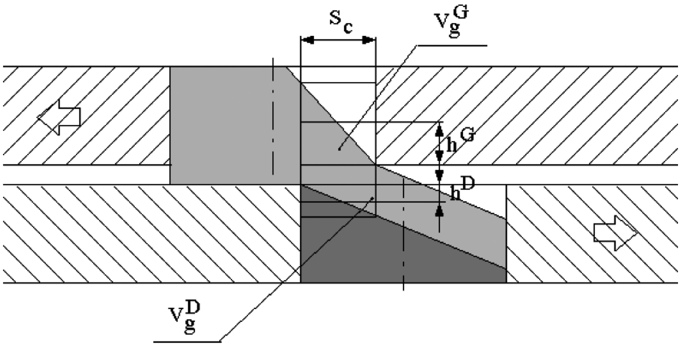

Fig. 4. Diagram of filling the quasi-cutting unit with biomass grains

When the hole is filled (Fig.4), the common part of the cross-sections, and pairs of transporting/grinding holes are reduced and the quasi-cutting process begins. An initial assumption may be made that each grain is subject to the disintegration process in the cross- section of the holes. The orientation of the grain with respect to the plane in which cutting occurs is random with even distribution. A grain of any length will be divided with equal likelihood into two smaller pieces of grain with the total length equal to the length of the grain before cutting.

The cutting occurs always in the material that, before filling, was in the previous disc and within area $S = S_c \left(V_n^m + V_{n+1}^m \right)$. The height of the material column above the cutting plane is:

$$h^G = \int_{cw(\alpha_c)} \min \left\{ y_n, H_{\vec{g}_G}(r) \right\} d^2 r \tag{7}$$

and below the cutting plane:

$$h^D = \int_{cw(\alpha_c)} \min \left\{ y_{n+1}, -H_{\vec{g}_D}(r) \right\} d^2 r \tag{8}$$

The likelihood that after cutting in the material below the cutting plane, there is a particle with the length within $(x, x+\,dx)$ range is:

$$\rho_n^m dx - \rho_n^m \frac{x}{h^D} + \int_{l_{\min}}^{l_{\max}} \frac{l}{h_D} \frac{dx}{l} \rho(l)\, dl \tag{9}$$

The first element corresponds to the pre-cutting likelihood. From this likelihood, one must deduce that an $x-$sized particle will be within the cutting area. This is expressed with the product of the likelihood that an x-sized particles and x/h^D relation occurs (the end of a particle must fit within section $(0, x) \subset (0, h^D)$). The third factor is integration from x to l_{max} of the likelihood that: $l(\rho(l)dl)$ long particle is found; it is within the cutting plane l/h^D, after cutting a particle with a length within a range of $(x, x+\,dx)$ (dx/l) is obtained.

On the basis of the above, one determines that the distribution of grain length in the material, that filled an empty space in $(n+\,1)$ disc changes as follows:

$$\tilde{\rho}_{n+1}^m(x) = A_{n,m} \rho_n^m = \left(1 - \frac{x}{h^D} \right) \rho_n^m(x) + \frac{1}{h^D} \int_x^{l_{\max}} \rho_n^m(l)\, dl \tag{10}$$

and correspondingly in the material remaining in n disc. It must be noted that the column of the material subject to cutting is not, in general, all the material poured to the bottom hole. Its volume equals $S_c \cdot h^D$, while the volume of all the material poured from the previous hole to the next hole equals $V^D_{\vec{g}}(\alpha_c) - V^m_n$, therefore

$$\tilde{\rho}_n^{m+1}(x) = \tilde{B}_{n,m}\rho_n^m = \left(1 - \frac{Sc\cdot x}{V^D_{\vec{g}}(\alpha_c) - V^m_{n+1}}\right)\rho_n^m(x) + \frac{Sc}{V^D_{\vec{g}}(\alpha_c) - V^m_{n+1}}\int_x^{l_{\max}} \rho_n(l)\,dl \tag{11}$$

The functions obtained are non-negative, as a sum of two non-negative elements. By integrating from 0 to $l$we can easily verify that those are the likelihood distributions:

$$\begin{gathered}\int_o^{l_{\max}} \tilde{\rho}^m_{n+1}(x)dx = 1 - \frac{\tilde{x}}{h^G} + \frac{1}{h^G}\int_o^{l_{\max}}\int_o^{l_{\max}} \rho_n(l)\,dldx = 1 - \frac{\tilde{x}}{h^G} \\ + \frac{1}{h^G}\int_o^{L_{\max}}\int_0^x \rho_n(l)\,dxdl = 1 - \frac{\tilde{x}}{h^G} + \frac{1}{h^G}\int_o^{l_{\max}} x\rho_n(l)\,dx = 1\end{gathered} \tag{12}$$

and similarly for distribution $\tilde{\rho}_n^{m+1}$. Therefore, operators $A_{n,m}$ and $\tilde{B}_{n,m}$ are properly specified stochastic operators.

For the purpose of simplification, an assumption has been made that after cutting the distribution of grain length in (n+1) disc is even (cut fraction and fraction already in the hole prior to cutting will mix); therefore, it is the weighted average of ρ^{m-1}_{n+1} and $\tilde{\rho}^{m-1}_{n+1}$.

$$\rho^m_{n+1}(x) = \frac{V^m_{n+1}}{V^D}\rho^{m-1}_{n+1} + \frac{V^D - V^m_{n+1}}{V^D}B_{n,m}\rho^m_n(x) \tag{13}$$

Filling of the quasi-cutting unit and therefore the efficiency of the cutting process depends on the values of function V^D, V^G and S_c, which depend on the direction of effective gravitation and the sum of the material volume in both holes before cutting $\left(V^m_{n+1} + V^m_n\right)$.

4. Evaluation of efficiency and working unit filling

It is difficult to determine filling of the quasi-cutting unit with biomaterial grains and other material. A decision was made to rely on the simplicity and advantages of the indirect method that involves conclusions based on the study of the mass efficiency. The study was conducted with 5-disc and 7-disc multi-hole grinders owned by the Department of Production Technology and Engineering of the University of Technology and Life Sciences in Bydgoszcz (Fig.1). The results obtained for the grinding process using a variable number of discs (2-7) and variable grain feeding method are shown in Table 4 and Table 5. Full feeding (marked as 1) is to be understood as the maximum setting of the screw feeder. Partial feeding (marked as 1/2) means such a setting of the feeder that stabilises feed material amount at a level of 50% of maximum feeding (full feeding). Target efficiency is defined as the weight of a desired screened fraction of the resulting product selected using dimensional criteria obtained with a specific time unit. For the purposes of this study, three fraction categories for ground material were applied: <0.8 mm; 0.8-1.6 mm;> 1.6 mm.

The number of drives (drive units: motor, transmission adequately configured) and the number of grinding discs (actively participating in the grinding process) in the working unit do not correspond to the proportionate increase in efficiency, e.g. in case of triticale grains, for full feeding (identified with 1).

Instantaneous efficiency, and indirectly also filling of the working unit with grains, are of a non-linear nature. The greatest increase in efficiency has been observed during grinding with only two discs and when the feeding was changed to partial (1/2) – only with a set of five discs. A similar situation occurred for maize grain grinding, where the greatest efficiency for full feeding (1) was obtained with drives for a four disc set, and with partial feeding (1/2), a greatest increase in efficiency was obtained only with a set of six grinding discs.

Table 4.

Maize grain grinding tests results

	No. of discs	Feeding amount		Grain diameter mm	Ground material fractions %			Target ground material efficiency g/s		
		1	1/2		<0.8 mm	0.8-1.6 mm	>1.6 mm	$Q_{<0.8}$	$Q_{0.8-1,6}$	$Q_{>1.6}$
Maize grains	2		x	8.2	6	16	78	0.03	0.08	0.39
	2	x		8.2	4.5	10	85.5	0.33	0.70	6.07
	3		x	8.2	6.2	9.4	84.4	0.02	0.03	0.27
	3	x		8.2	4.2	11.5	84.3	0.20	0.45	3.43
	4		x	8.2	12.3	22.8	64.9	0.07	0.13	0.37
	4	x		8.2	11.2	21.6	67.2	1.45	2.80	8.70
	5		x	8.2	10.4	27.6	62	0.03	0.08	0.18
	5	x		8.2	11.3	25.4	63.3	1.355	3.05	7.60
	6		x	8.2	14	38.5	47,5	0.17	0.47	0.58
	6	x		8.2	12	36.4	51.6	0.90	2.73	3.87
	7		x	8.2	17.2	35.7	47,1	0.12	0.25	0.33
	7	x		8.2	12.4	33.9	53.7	0.73	2.00	3.17

The analysis of target efficiency $Q_{<0.8}$ (fraction with a size of > 0.8 mm) has shown that filling of the grinding unit and efficiency obtained with a set of two or three discs when grinding triticale grains is very low, i.e. it reaches only (0.44-1)% (in relation to mass efficiency Q), while for maize grains, it is much higher (4.9-5.6)%. As the number of discs is changed, e.g. to more than four, both for grinding maize and triticale, much greater values of filling and efficiency have been observed- even 10 times greater for triticale and 2.5 times greater for maize. The degree of triticale and maize feeding has no significant influence on an increase in target efficiency ($Q_{<0.8}$). Therefore, the number of active, driven, used discs of the working unit significantly affects an increase in target efficiency ($Q_{<0.8}$), unlike the feeding method.

Table 5.

Triticale grain grinding tests results

	No. of discs	Feeding amount		Grain diameter mm	Ground material fractions %			Target efficiency g/s		
		1	1/2		< 0.8 mm	0.8-1.6 mm	> 1.6 mm	$Q_{<0.8}$	$Q_{0.8-1.6}$	$Q_{>1.6}$
Triticale grains	2		x	3.5	0.4	3.1	96.3	0.02	0.12	3.70
	2	x		3.5	0.4	1.7	97.9	0.20	0.90	51.50
	3		x	3.5	1.5	6	92,5	0.07	0.30	4.57
	3	x		3.5	1	3,6	95,4	0.40	1.35	36.05
	4		x	3.5	3.9	19.2	76.9	0.20	1.00	4.00
	4	x		3.5	3.3	13	83.7	1.40	5.60	36.10
	5		x	3.5	3.5	18.2	78.3	0.53	2.73	11.74
	5	x		3.5	3.4	12.6	84	1.40	5.30	35.30
	6		x	3.5	4.8	35.2	60	0.60	4.40	7.50
	6	x		3.5	4.1	24.5	71.4	1.80	10.80	31,50
	7		x	3.5	4.8	29.4	65.8	0.40	2.47	5.53
	7	x		3.5	5.1	27.6	67.3	2.15	11.50	28.05

Efficiency and filling with target fraction in the product size range (0.8, 1.6) mm, $Q_{0.8-1.6}$ as compared to efficiency and filling with grains (grinding product smaller than 0.8mm: $Q_{<0.8}$) during grinding of maize and triticale grains, with two and three discs, are even six times greater. The next significant increases of efficiency and filling (two and three time increase) both for maize and triticale grains occur when four discs are used for the grinding process. Then, efficiency and filling with target size grains $Q_{0.8-1.6}$ (85-100) % can be observed when the number of discs is increase to six. Another exception has also involved the study of triticale grains during full feeding

(1), where a significant decrease in efficiency and target filling has been observed (from 27.58 to 21.68) %.

The diagram of efficiency and filling with target grains larger than 1.6mm: $Q_{>1.6}$, is completely different, since the efficiency with two or three discs used reaches very high levels (within (78-97.9)% of mass efficiency); however, as the number of discs used increases, efficiency and target filling $Q_{>1.6}$ decrease. A significant decrease in efficiency and filling occurs when a set of four discs is used for the grinding process. Then, a decrease in efficiency and filling is noticeable when a set of six discs is used. Unlike in previous studies, the feeding degree set to partial (1/2) resulted in a decrease in efficiency and filling with target grains: $Q_{>1.6}$.

Efficiency and filling with target grain material reached twenty percent $Q_{20\%<1.6}$ only when a set of four discs was used to grind maize (regardless of the feeding method) and triticale only with partial (1/2) feeding. For full feeding (1) of triticale, efficiency and filling with target material $Q_{20\%<1.6}$, was obtained with a set of six discs with quasi-cutting holes. Also this time, for maize and triticale, partial feeding (1/2; 50%) was set, which contributed to an increase in efficiency and filling with target grains: $Q_{20\%<1.6}$. It must be noted that despite low values of efficiency and filling with target grains for maize (in particular during the partial feeding) and triticale, they reached the highest values, exceeding even 50% of fed material.

5. Results analysis

An analysis of the results obtained indicates interrelations within the system consisting of the described quasi-cutting unit, grain biomass, filling of the transporting/grinding space and target efficiency of the process.

In the first approach to the model of those interrelations, a statement can be made that filling of the quasi-cutting unit, and therefore the efficiency and even effectiveness in its broad sense of the process used to cut biomass grains, depend on values of function V^D, V^G and S_c, which depend on the direction of motion/drive gravitation of the grinding unit and the sum of volume of the material in both holes (constituting a unit) prior to cutting $(V^m_{n+1} + V^m_n)$.

In the second approach to the model, i.e. verification under mechanical conditions: drive, quantities of maize, interrelations grains at various stages of the grinding/transporting process of the working grinder discs – depend on the observed, measurable, known and specified volumes V^G, V^D of the previous and the next hole and S^G, S^D of the cross-section surface area of the previous and the next hole.

In the practical approach related to construction and operation of machines, including grinding machines, a new pro-developmental area of solutions may be proposed for further analyses of an integrated grinding system in the field of the variability of design features $C_k \in \Phi$. To this end, one uses a design solution as a logical conjunction of the criteria and possible (within the permissible area) design features of the quasi-cutting unit – within the conceptual space, being the optimum solution from the point of view of the specified criteria including: purpose, energy minimum, self-regulation and level structure.

The method and design of the drive transmission to the working unit of the grinder, particularly in the case of innovative multi-hole, multi-disc solutions (based on the Polish patent [4]), significantly affect selected properties and quantities typical of the process including filling and efficiency of the target product. The multi-disc grinding process enables precise determination of values of the physical quantities, postulated in a given reality, such as gradients of rotational speed of adjacent working discs/edges for specific material and functional features. A failure to fulfil the above results in a conventional grinding process, i.e. without filling and efficiency control.

6. Conclusions

Taking into consideration variables of biomass grain feeding and variables of the working unit, it is possible to develop adequate mathematical models required for and supporting the design process of multi-disc grinders. The form of well-defined models determines their optimal construction and selection from a range of permissible design features of elements of multi-disc grinding units, in particular: efficiency and unit energy consumption.

Mass efficiency of the multi-edge grain grinding is directly dependent on the filling level of the feeding and grinding units. These basic process parameters, according to analysed mathematical (model) and statistical (efficiency functions, filling) dependencies of biomass grains, are significantly influenced by:

1) Design features of the working space limited to: volume and cross-sections of adjacent holes,

2) Kinematic parameters (angular speed of even discs – ω_0, and odd discs – ω_1), process parameters of discs and edges of the holes ($Q_{<0.8}$, $Q_{0.8-1.6}$, $Q_{>1.6}$ – in target percentage shares).

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Model wypełnienia zespołu roboczego, wielotarczowego rozdrabniacza ziaren biomasy

Streszczenie

Istotny wpływ na wydajność, funkcjonalność i sprawność procesu rozdrabniania ziaren zbóż mają trudne do opisania zjawiska, zachodzące w przestrzeni roboczej rozdrabniacza. W maszynowym wielotarczowym rozdrabnianiu biomateriałów ziarnistych, należy tak dobrać konstrukcję zespołu

quasi-ścinającego, objętości, przekroje otworów transportowo-rozdrabniających, ich ruch i cechy konstrukcyjne tarcz (zespołu rozdrabniającego), aby zostały zrealizowane założone funkcje: rozdrabnianie w przestrzeni międzyotworowej (przy minimalnej energochłonności procesu i maksymalnej wydajności) oraz minimalizacja zjawisk niepożądanych związanych z mieszaniem i transportem.

Warunkiem optymalizacji konstrukcji zespołu quasi-ścinającego jest model matematyczny. Wśród wielu warstw problemu, w pracy przedstawiono przykładowe postępowanie, prowadzące do modelu wypełnienia zespołu quasi-ścinającego ziarna biomasy wraz z wstępną jego weryfikacją w warunkach badania wydajności rozdrabiania ziaren kukurydzy i pszenżyta na innowacyjnym rozdrabniaczu wielootworowym, 5-cio i 7-mio tarczowym.