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HILBERT-TWIN – A NOVEL HILBERT TRANSFORM-BASED METHOD TO COMPUTE THE ELASTIC MODULUS IN HIGH-RESOLUTION MECHANICAL SPECTROSCOPY HRMS

HILBERT-TWIN – NOWA METODA OBLICZEŃ MODUŁU SPRĘŻYSTOŚCI W WYSOKOROZDZIELCZEJ SPEKTROSKOPII MECHANICZNEJ HRMS

A novel Hilbert-twin (H-twin) method is introduced as an alternative method for the computation of the resonant frequency for exponentially damped free decays embedded in noise. We also present the comparison among the following methods used to compute the dynamic elastic modulus in solids: the parametric OMI (Optimization in Multiple Intervals), the Yoshida-Magalas (YM) interpolated discrete Fourier transform, the Hilbert-twin (H-twin), and discrete Fourier transform (DFT) methods. It is concluded that the OMI and YM methods are the best methods to compute the elastic modulus from discrete exponentially damped free-elastic decays embedded in unavoidable noise.

Keywords: elastic modulus, mechanical spectroscopy, Hilbert transform, instantaneous frequency, interpolated discrete Fourier transform, Fourier transform

W pracy przedstawiono nową metodę Hilbert-twin, którą opracowano do obliczeń częstotliwości rezonansowej wykładniczo tłumionych harmonicznym sygnałom odkształceń sprężystych próbki zawierających szum. Jest to pierwsza praca, która uwzględnia obecność i wpływ szumu na wyniki obliczeń modułu sprężystości metali i stopów metali.

Porównano i przeanalizowano wyniki obliczeń dynamicznego modułu sprężystości uzyskanego z kilku metod: metody parametrycznej OMI (Optimization in Multiple Intervals), metody Yoshida-Magalas (YM) opartej na interpolowanej dyskretnej transformacji Fouriera, metody Hilbert-twin (H-twin) oraz dyskretnej transformaty Fouriera (DFT). Z przeprowadzonych badań wynika, że metody OMI i YM są najlepszymi metodami estymacji modułu sprężystości z dyskretnych sygnałów odkształceń sprężystych materiałów (wykładniczo tłumionych drgań swobodnych) zawierających szum.

1. Introduction

For exponentially damped time-invariant harmonic oscillations embedded in experimental noise, $\varepsilon_w(t)$, the logarithmic decrement, δ , and the resonant frequency, f_0 , can be determined from Eq. (1) using the digitized data $A_i(t)$ and t_i of free decaying oscillations:

$$A(t) = A_0 e^{-\delta f_0 t} \cos(2\pi f_0 t + \phi) + \varepsilon_w(t) + dc, \quad (1)$$

where A_0 is the maximal strain amplitude, t is a continuous time in seconds, $-\pi < \phi \leq \pi$ is the phase in radians, and dc is a system-dependent offset or the slow-varying trend. Deployment of the center of free decaying oscillations, referred to as the Zero-Point Drift (ZPD), is neglected [1]. $\varepsilon_w(t)$ is signal noise characterized in this work by the signal-to-noise ratio, $S/N = 60$ dB. The term 'free-elastic decay', introduced by the authors, emphasizes the presence of the experimental noise in the time series of free decaying data used to estimate the dynamic elastic modulus and dissipation of mechanical energy in solids induced e.g. by plethora of relaxation phenomena.

Fourier transform is a traditional method in the time-frequency analysis of linear, stationary and global time

series. In mechanical spectroscopy, Fourier analysis is used in the time-frequency analysis to detect frequency components in free-elastic decaying oscillations recorded in the time-domain. The main resonant frequency, f_0 , of the signal $A(t)$ is estimated in the frequency domain from the discrete Fourier transform (DFT) [2] and the interpolated discrete Fourier transform (IpDFT) [2-5]. The resonant frequency, f_0 , can also be estimated in the time-domain from the OMI (Optimization in Multiple Intervals) [1-3,6-8], the novel Hilbert-twin (H-twin), and the zero crossing methods [6,8].

In this paper, we report the first results obtained from the H-twin method. We also compare the performance of the H-twin method to the OMI and integral transform-based methods.

2. The Hilbert-twin – A novel Hilbert transform-based method to estimate the resonant frequency and the elastic modulus from free decaying oscillations embedded in noise

Let us define the complex analytic signal for free-elastic decaying oscillations, $A(t)$:

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$$z(t) = A(t) + i \hat{A}(t), \quad (2)$$

where $\hat{A}(t)$ is the Hilbert transform of the $A(t)$. The instantaneous amplitude of the analytic signal is given by

$$a_H(t) = \sqrt{A(t)^2 + \hat{A}(t)^2}. \quad (3)$$

The instantaneous phase, $\varphi_{inst}(t)$, can be determined from:

$$\varphi_{inst}(t) = \text{arctg} \left(\frac{\hat{A}(t)}{A(t)} \right). \quad (4)$$

The instantaneous phase, $\varphi_{inst}(t)$, of free decaying oscillations is a function of time. The instantaneous frequency, $f_{inst}(t)$, is defined by the time derivative of the instantaneous phase, given by Eq. (4). Thus the instantaneous frequency, $f_{inst}(t)$, [9-12] is:

$$f_{inst}(t) = \frac{1}{2\pi} \frac{d}{dt} \varphi_{inst}(t) = \frac{1}{2\pi} \frac{d}{dt} \text{arctg} \left(\frac{\hat{A}(t)}{A(t)} \right). \quad (5)$$

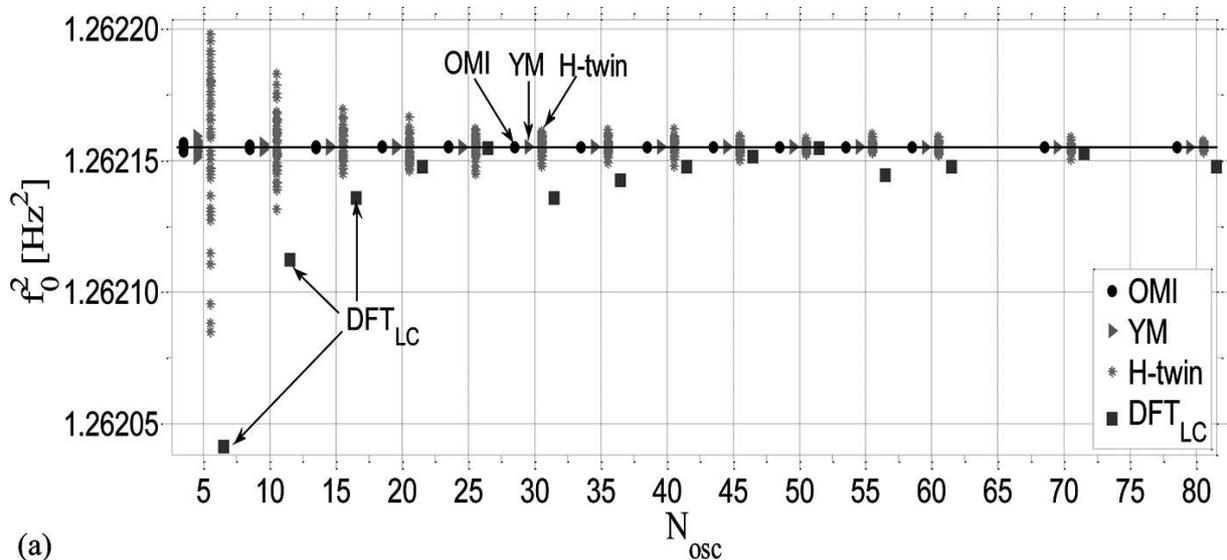
The locations of t for $A(t) = 0$ are determined from the instantaneous frequency, $f_{inst}(t)$, which corresponds to the peak maximum for each half-period. The $f_{inst}(t)$ straight line can be estimated using a linear least-squares fit procedure to compute the zero value $A(t) = 0$ and the time spacing between zero crossing points; each half of a period is estimated in the analyzed signal. Thereby the mean resonant frequency for the entire free-elastic decay, f_0 , becomes available (the dynamic elastic modulus measured in a resonant mechanical spectrometer is $\sim f_0^2$). This method refers to as the 'Hilbert-twin phase crossing', which requires signal preprocessing, that is, a twinning procedure of the experimental signal $A(t)$. In this approach, the instantaneous and resonant frequencies are estimated in the time-domain over the duration of free decaying oscillations embedded in white noise.

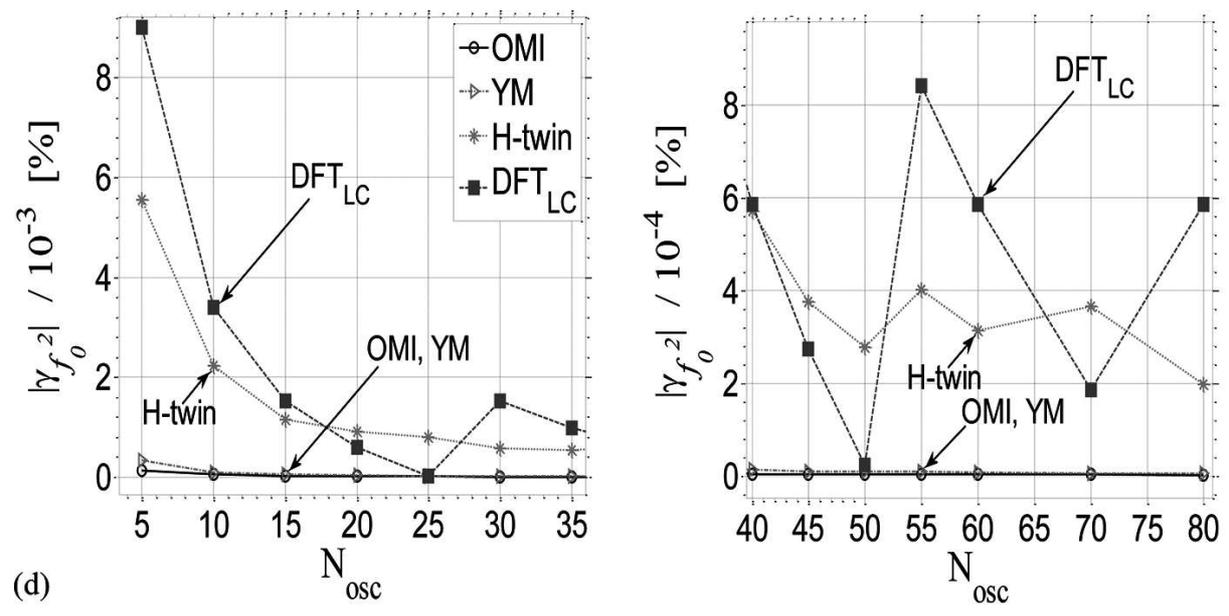
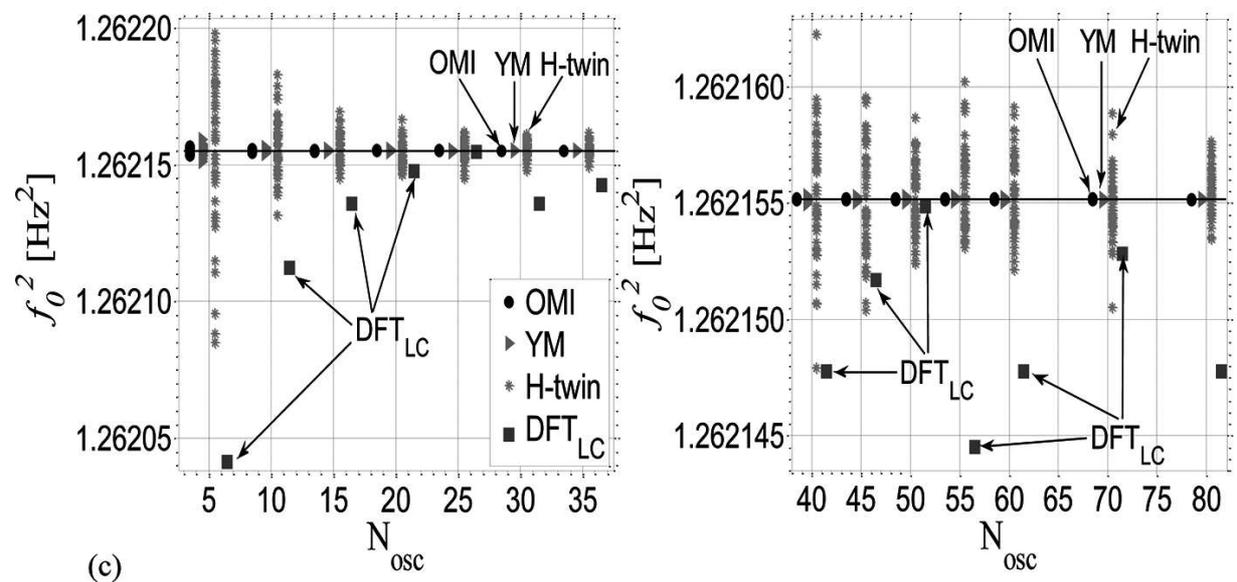
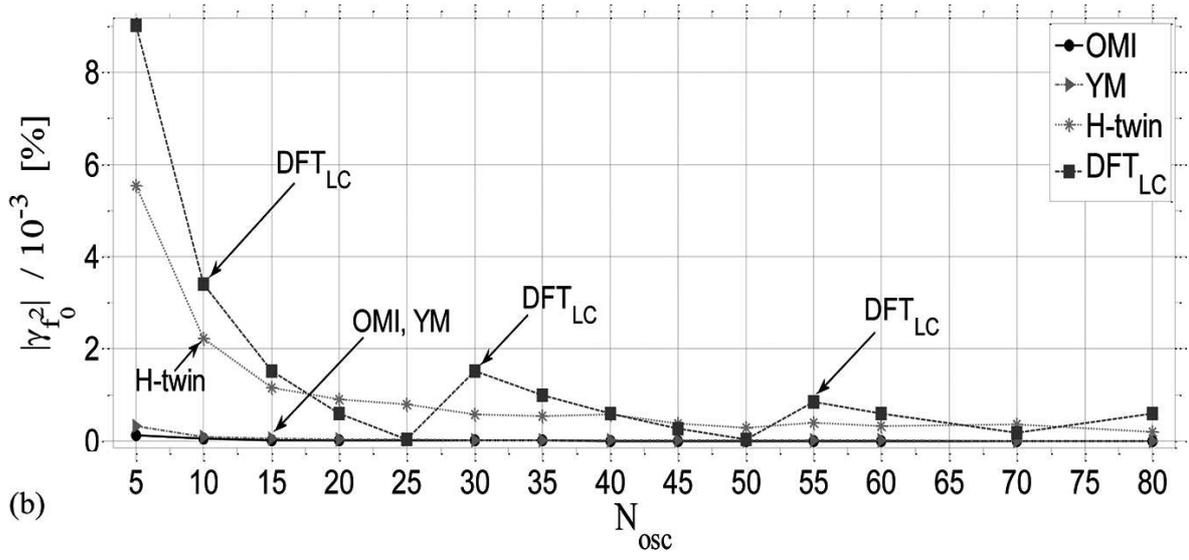
In this paper we compare the performance of the following four computing methods to estimate the resonant frequency, f_0 : (1) the parametric OMI method, (2) the Yoshida-Magalas (YM) method based on interpolated discrete Fourier transform, (3) the Hilbert-twin (H-twin) method, and (4) the classic DFT method.

DFT algorithms are available in numerous commercial software packages. It is well known that the Fourier transform has problems with its resolution. The relative error in estimation of the dynamic elastic modulus ($\sim f_0^2$) by DFT method is around 2-5% for damped harmonic oscillations recorded in a low-frequency torsion pendulum [3]. The major source of the error arises from the truncation of the Fourier series for the time-domain function and the bias introduced by the leakage. The truncation will cause the elastic modulus to be under-estimated whereas the leakage will cause the elastic modulus to be over-estimated. The estimation error depends on the duration of the measurement and the sampling frequency [2,3]. The DFT method is robust against noise. High estimation error (up to 5%) is satisfactory for less demanding applications. Nevertheless, DFT methods yield poor estimation of the elastic modulus. This conclusion is important because frequency analyzers routinely use the DFT technique to estimate the resonant frequency. Thus one must contend with uncertainties of the resonant frequency estimated by hardware solutions. This level of estimation error must be *a priori* accepted by experimentalist. To alleviate this limitation, conventional DFT method was tailored to experimental conditions encountered in an inverted torsion pendulum [1,2], herein referred to as the DFT_{LC} [2] (see Fig. 1.)

We should emphasize that even a 1°C change in temperature could give rise to a noticeable change in the elastic moduli of a metallic sample. This is why advanced studies of pinning stages in irradiated metals [13,14], interaction between dislocations and foreign interstitial atoms in freshly deformed Fe-C alloys [15-17], and dislocation blocking by Cottrell atmospheres of foreign interstitial atoms in bcc alloys [18-22] call for much higher resolution in fine variations of the elastic modulus. This is why the resolution/sensitivity in the estimation of dynamic elastic modulus is critical in high-resolution mechanical spectroscopy, HRMS.

In the following we show that the H-twin method provides better estimation of the modulus compared to DFT method and carefully designed DFT_{LC} algorithm optimized for free-elastic decaying signals acquired in a low-frequency resonant mechanical spectrometer (inverted torsion pendulum).





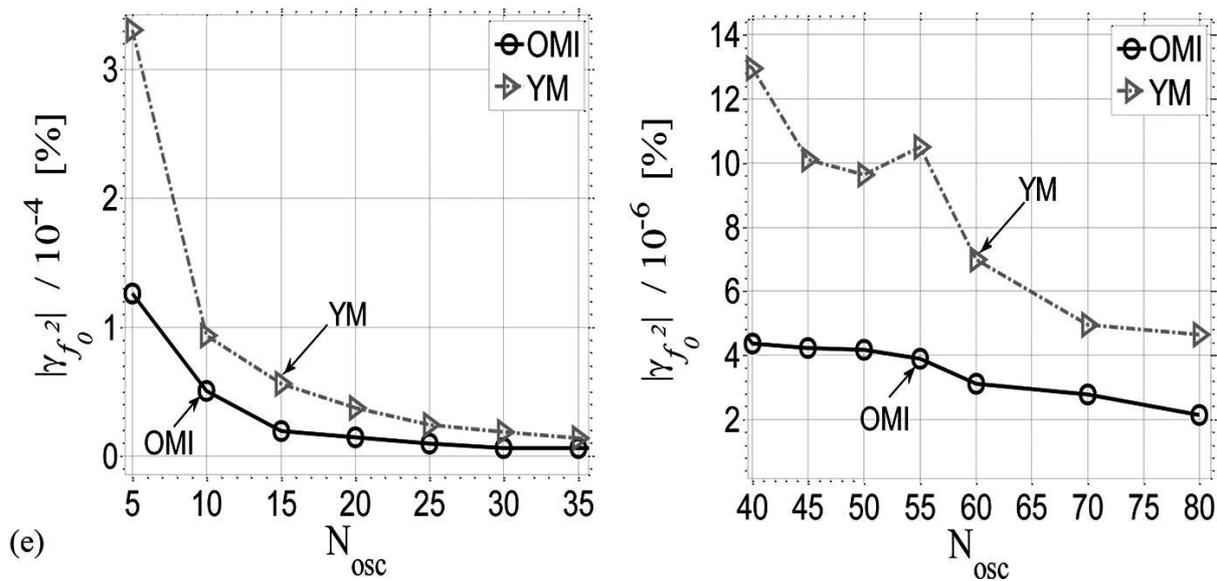


Fig. 1. Estimation of the dynamic elastic modulus according to different computing methods: OMI, YM, H-twin, and DFT_{LC} for 100 decays. (a) Variation of the modulus as a function of the number of oscillations N_{osc} . (b) Variation of the maximal relative error of the modulus $|Y_{f_0}^2|$ as a function of the number of oscillations N_{osc} . (c) A zoom on Fig. 1a for different durations of free-elastic decays. (d) and (e) A zoom on Fig. 1b. The logarithmic decrement, $\delta = 0.0005$, the resonant frequency $f_0 = 1.12345$ Hz, the sampling frequency $f_s = 4$ kHz, and the signal-to-noise ratio, $S/N=60$ dB. (For interpretation of the references to color in this figure legend, readers are referred to the web version of this article.)

Figure 1 illustrates the performance of the OMI, YM, H-twin and DFT_{LC} methods as a function of the number of oscillations N_{osc} (the number of periods) for free-elastic decays characterized by the signal-to-noise ratio, $S/N = 60$ dB. The DFT_{LC} method underestimates the elastic modulus [3] (Figs. 1a, 1c) and generates the highest maximal estimation error $|Y_{f_0}^2|$ (Fig. 1b). The DFT_{LC} method properly estimates the elastic modulus for few points only ($N_{osc} = 25, 50$; see Figs. 1c, 1d). This effect can be accounted for by the coherence condition between the analyzed free-elastic decaying signal and the sampling frequency.

It is obvious that the H-twin method (Hilbert-twin phase crossing) performs better than DFT (Figs. 1a-1d). The drawback of applying the H-twin method lies, however, on larger dispersion of computed modulus values compared to the OMI and YM methods (Figs. 1c, 1d).

Figure 1d demonstrates that the YM and OMI methods are characterized by very small estimation error of the order of $10^{-5}\%$ [2,3,6-8] (in the presence of noise, $S/N=60$ dB). Although the estimation error depends on duration of free-elastic decaying oscillations [6] and the sampling frequency [3,7], the performance of the OMI and YM methods entirely fulfills requirements of high-resolution mechanical spectroscopy, HRMS.

The OMI, YM, H-twin, DFT, and zero crossing methods are collected in [22].

3. Conclusions

It is demonstrated that the OMI and YM methods are the best to compute the elastic modulus from exponentially damped free-elastic decays embedded in unavoidable noise. The results here are used primarily to highlight new methods for the estimation of the elastic modulus. The main takeaway

of the present paper is the potential of using the OMI and YM methods as the 'gold standard' in high-resolution mechanical spectroscopy, HRMS and classical resonant mechanical spectrometers.

The novel Hilbert-twin method is proposed based on the Hilbert transform of free-elastic decaying oscillations subjected to signal preprocessing to eliminate the oscillatory behavior of envelope (i.e., ripples). The H-twin method outperforms traditional DFT methods, which are widely used to estimate dynamic elastic modulus of solids in low- and high-frequency resonant mechanical spectrometers.

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