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JITTER MEASUREMENT ON THE BASIS OF HIGH-PRECISION EVENT TIMER

Yuri Artyukh, Eugeny Boole

Institut of Electronics and Computer Science, 14 Dzerbenes Str., LV-1006 Riga, Latvia (✉ artyukh@edi.lv)

Abstract

Currently the high-precision event timers represent powerful tools for time measurement in various applications, including jitter measurement. Applied potential of this technology is illustrated by the example of clock jitter measurement and analysis based on the application of a high-precision event timer. The basic measurement procedures resulting in estimations of commonly used jitter parameters (such as accumulated jitter, period jitter, clock-to-clock jitter) are discussed. An approach to informal interpretation of statistical jitter characteristics based on theoretical jitter model and results of computer simulation is offered. Experimental results of jitter measurement and analysis for high-precision clock oscillators confirm the assumption that currently the event timing can provide for jitter measurement precision comparable with traditional oscilloscope-based techniques.

Keywords: jitter measurement, event timer, accumulated jitter, statistical jitter analysis.

1. Introduction

As known, there are two basic approaches to jitter measurement and analysis: in the frequency domain and in time domains. Jitter measurement in the frequency domain is initially related to spectrum analysis, resulting in a continuous phase noise plot over a range of frequencies. Then phase jitter is calculated as the integration of phase noise over a certain spectrum [1, 2]. As for jitter measurement in the time domain, basically it is performed using high-performance real-time oscilloscopes with sampling rate up to tens of GHz, and special software that calculates where in time the signal crossed the preset level between the digitized points [3]. Usually in this case the time intervals between two events (so-called delta-times) are measured first, and other timing parameters are derived from such measurement. The main advantage of this approach is the ability to directly handle high-frequency signals (in the GHz range) with high delta-time precision. However as applied to the lower signal frequencies such approach seems too complicated in implementation and too expensive.

Recently, along with this technique, the jitter measurement on the basis of so-called event timers (frequently termed time digitizers) came into use [4]. Unlike the real-time oscilloscope, the event timer directly measures the time at instants when some events occur. In jitter measurement some predetermined points of the input signal (such as edges of input pulses or signal zero crossings) are considered as events; next the timing of these events provides full data in digital form about their actual position on the time axis. In this respect conceptually the event timers are ideally suited to direct jitter measurement and its further digital analysis. As for performance of the event timers, basically it is specified by precision/speed ratio that varies in a wide range. The highest-speed event timers (such as 9353 100-ps Time Digitizer from ORTEC) offer maximum measurement rate up to 1 GHz and precision of about of 85 ps

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RMS whereas the highest-precision event timers offer a precision about of a few picoseconds but at much lower measurement rate [5].

Applied possibilities of the present-day event timers for jitter measurement and analysis are not sufficiently investigated. In particular, there is indefiniteness in specification of the jitter through event timing, in restrictions for analyzing precision in view of limited event timer resolution, etc. We shall discuss part of these problems by example of random jitter measurement and analysis for clock oscillators with the use of a high-precision event timer.

2. Technique of jitter measurement by an event timer

By definition the event timer supposes detecting the events before their measurement. In view of that a special block (signal conditioner) is used for generating these events when the input signal crosses a preset level, i.e. when the signal phase is being incremented by one period. Usually the events are presented in the form of normalized pulses arriving at the timer input. If necessary, additionally this block reduces the frequency of events to match it with available measurement rate of the event timer. It is assumed that the conditioner is carefully designed and does not introduce significant distortions into the original jitter of the signal.

In response to the input events, the event timer generates a series of digital time-stamps $\{t_k\}_0^N$ that reflect actual incrementing of time instants when the signal phase is being incremented by one period (Fig. 1).

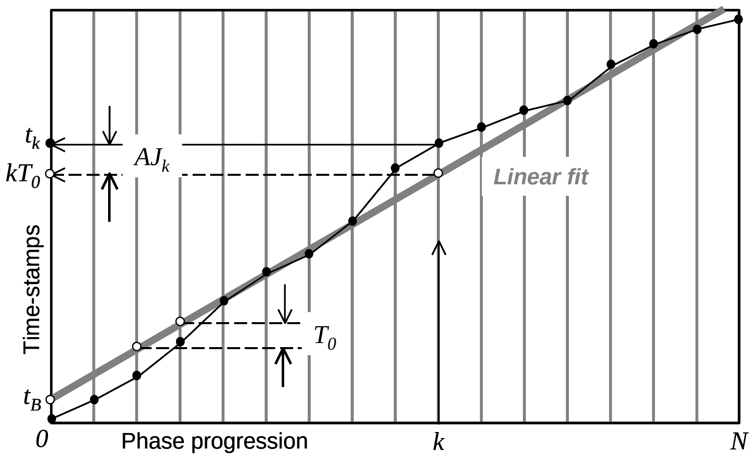

Fig. 1. Principle of jitter measurement through event timing.

Although most event timers offer actually an unlimited measurement range, in jitter measurement usually the cycle duration does not exceed hundreds of milliseconds inasmuch as by definition the jitter implies phase oscillations at frequencies greater than 10 Hz [6].

Deviation of the actual incrementing of the time-stamps relatively to their ideally linear incrementing represents dynamics of jitter accumulation. In this case jitter accumulated at every k-th period of the input signal can be calculated as residuals of time series data:

$$\{AJ_k = t_k - (kT_0 + t_B)\}^N, \qquad (1)$$

where $(kT_0 + t_B)$ is a linear function that has the best fit to a series of time-stamps, and N – total number of the time-stamps obtained in a single measurement cycle. In this case T_0 is an averaged period of the input signal, and t_B is the reference time for a particular cycle of measurement. Hereafter jitter determined in such a manner, is referred to as A-jitter.

Note that correct choice of the fitting method considerably influences A-jitter determination. To be specific, we perform such fitting on the basis of the least-square method

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that is frequently used in the linear regression approach, and separately for each measurement cycle. In this case the *A*-jitter of a signal (if it is sufficiently stationary) can be evaluated repeatedly with any time-gaps, and without substantial interference from its long-term instability. Additionally such fitting provides for cyclical precise measurements of the period T_0 to determine its deviation vs. real time if required.

A-jitter values evaluated in this way, arranged in line with period number increasing, represent a uniformly sampled *A*-jitter function. Notice that any integer division of the signal frequency results in reduction of the sampling rate, but does not introduce essential distortions in presentation of *A*-jitter function, i.e. it simply will be presented by a smaller amount of samples.

Using the *A*-jitter function, other conventionally used jitter characteristics can be simply derived. Specifically, according to the commonly used metrics, the first differences

$$\{PJ_k=AJ_k\text{-}AJ_{k\text{-}1}\}^N, \tag{2}$$

conform to so-called period jitter (hereafter referred to as *P*-jitter), and the second differences

$$\{CJ_k=PJ_k\text{-}PJ_{k\text{-}1}\}^N, \tag{3}$$

conform to so called cycle-to-cycle jitter (hereafter referred to as *C*-jitter). In the same manner the differences of higher order can be calculated if desired.

Let us notice that there are a lot of different definitions of the jitter and its particular metrics. For example, by general definition given in the ATIS Telecom Glossary, jitter is “the short-term variations of the significant instants of a digital signal from their ideal positions in time” [6]. In our case of jitter measurement “ideal positions” of these significant instants are their imagined positions on a linear fitting of their real positions. However, using this approach, it should be taken into account that such fitting, as applied to every measurement cycle, does not allow for possible natural trends in random jitter accumulation. Thus we always attribute such trends to long-term instability of clock sources what may result in understated estimation of real jitter as compared to other techniques of jitter measurement.

3. Experimental setup

To study the actual applied potential of this approach to jitter measurement and analysis, an experimental setup of a jitter Analyzer has been created (Fig. 2). The core of this Analyzer is represented by the Event Timer A033-ET which currently is widely used for high-precision time-of-flight measurements in Satellite Laser Ranging [7]. The A033-ET supposes the use of an external high-performance frequency standard and in our setup it is the “Thuderbolt GPS disciplined clock” from “Trimble Navigation Limited”.

A distinguishing feature of the A033-ET is high performance characteristics (in terms of precision/speed ratio) combined with reasonable unit price. In particular, the typical precision of the A033-ET for measurement of time intervals between two events is specified in the range of 3.5 to 4 ps RMS, implying RMS precision of a single time measurement approx. 2.3-2.8 ps. Let us note that such precision is quite comparable with the precision of time measurement for jitter analysis on the base of high-performance real-time oscilloscopes.

As for the measurement rate, the A033-ET supports a 20 MSPS burst rate for sequences of up to 2 600 events and a 12.5 MSPS burst rate for sequences of up to 16 000 events; the maximum average rate of a continuous (gapless) event measurement over a long period of time is 30 KSPS. To be applicable for measurement of higher frequencies, the Analyzer is supplied by a controllable prescaler (counter-divider) that allows operating at input signal frequencies up to approx 300 MHz without inserting significant distortions into jitter of the original signal. Generally the present-day microwave prescalers allow increasing the upper frequency, if necessary.

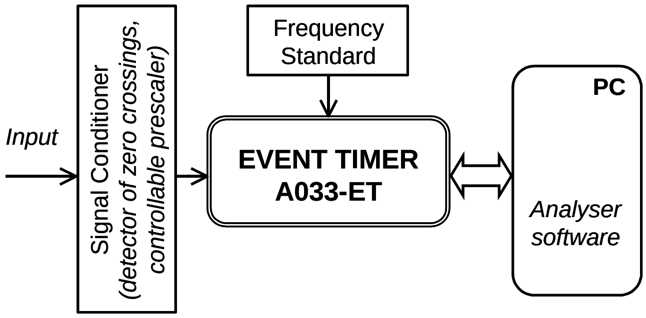

Fig. 2. Simplified block diagram of Jitter Analyzer.

The A033-ET is a PC-based instrument that allows application of software tailored to specific needs. In our case it was experimental software for jitter analysis running under MS Windows. According to the above technique of jitter measurement, in single-shot mode of operation this software provides calculating and displaying three basic jitter functions (Fig. 3). Additionally, in this mode the Analyzer calculates squared values of RMS deviations S_A^2, S_P^2 and S_C^2 for *A*-jitter, *P*-jitter and *C*-jitter respectively.

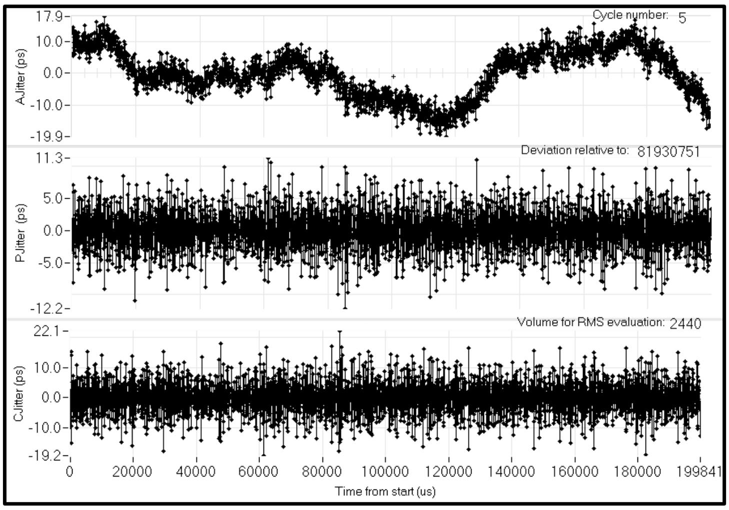

Fig. 3. Crystal clock generator jitter vs. time (1000 clock pulses).
From top to bottom: *A*-jitter, *P*-jitter and *C*-jitter.

Generally such jitter functions offer many useful data for qualitative jitter analysis, especially the *A*-jitter function, visually reflecting the particular process of jitter accumulation. For example, Fig. 3 demonstrates its noticeable deviation vs. time in comparison with other kinds of jitter. Similarly to this example, deterministic jitter in the form of repeatable periodic modulations of the jitter functions may be visually detected.

Along with the single-shot mode, the Analyzer supports the mode of repetitive jitter measurement. In this mode the Analyzer calculates and displays basic statistical characteristics of measured jitter from cycle-to cycle, indicating them as functions of the time at which every measurement cycle is performed (Fig. 4).

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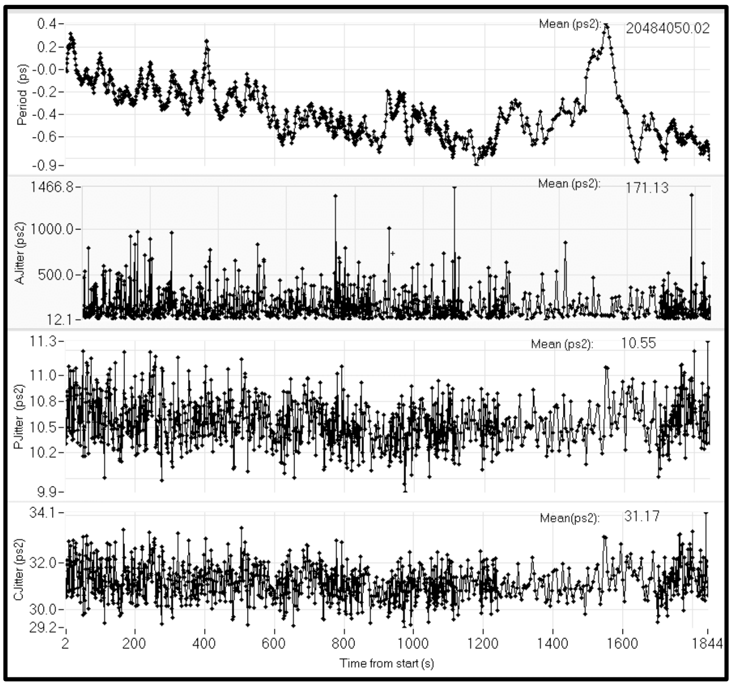

Fig. 4. Generalized characteristics of crystal clock generator jitter from cycle-to-cycle. From top to bottom: signal period deviation, squared RMS deviations of *A*-jitter, *P*-jitter and *C*-jitter respectively. Observation time: 30 min.

This Analyzer's mode permits to detect spurious bursts in jitter measurements and to exclude them from analyzing if desired. Additionally the Analyser provides for averaging of the S_A^2, S_P^2 and S_C^2 values to increase their reliability in statistical terms. As Figure 4 demonstrates, that can be especially necessarily for the *A*-jitter where dispersion of S_A^2 estimates from cycle-to-cycle can be very large.

4. Statistical clock jitter analysis

4.1. Theoretical model of randomly jittered oscillator

To better understand the physical essence of the above-mentioned statistical characteristics, let us consider often used simplified theoretical model of jitter accumulation [8, 9]. In particular, let us suppose that oscillator's clock jitter contains only random components so that the measured in every measurement cycle *k*-th time-stamp (k=0, 1, … N) measured in every measurement cycle can be presented as follows:

$$t_k=kT_0+\sum_{i=1}^{k}\Delta_i^A+\Delta_k^S, \tag{4}$$

where Δ_i^A - accumulative jitter component and Δ_k^S - superimposed (non-accumulative) jitter component. Usually the last one is caused by short-term instability of signal delay by buffer electronics of clock oscillators and, additionally, can include the measurement error.

Under these assumptions and taking into account equations (1-3), the jitter values AJ_k, PJ_k and CJ_k for every *k*-th time-stamp can be expressed as follows:

$$AJ_k=\sum_{i=1}^{k}\Delta_i^A+\Delta_k^S; \tag{5}$$

$$PJ_k=\Delta_k^A+(\Delta_k^S-\Delta_{k-1}^S); \tag{6}$$

$$CJ_k = (\Delta_k^A - \Delta_{k-1}^A) + (\Delta_k^S - 2\Delta_{k-1}^S + \Delta_{k-2}^S). \tag{7}$$

Let us assume that Δ_i^A and Δ_k^S are uncorrelated random values characterised by variances Var(A) and Var(S) respectively. In this case the variances of the above jitter values will conform to:

$$\mathrm{Var}(AJ_k) = k\mathrm{Var}(A) + \mathrm{Var}(S); \tag{8}$$

$$\mathrm{Var}(PJ_k) = \mathrm{Var}(A) + 2\mathrm{Var}(S); \tag{9}$$

$$\mathrm{Var}(CJ_k) = 2\mathrm{Var}(A) + 6\mathrm{Var}(S). \tag{10}$$

As can be seen from equations (8-10), the relative influence of the jitter components in total jitter of each jitter kind significantly differs. In particular, the accumulative component more pronouncedly is presented in A-jitter but the superimposed component – in C-jitter. Generally the relative influence of the superimposed component on total jitter rises according as the order of A-jitter differences increases.

4.2. Experimental evaluation of the model parameters

Let us consider possibilities to evaluate the variances Var(S) and Var(A) of the theoretical model on the basis of measurement data with the aim to predict the jitter characteristics for a particular clock oscillator under various conditions of its possible application.

As can be seen from equations (9, 10), the variances Var(PJ_k) and Var(CJ_k) are independent from the serial number k of the signal period. Correspondingly the values of S_P^2 and S_C^2 can be considered as experimental estimates of these variances, i.e. $S_P^2 \cong$ Var(PJ_k) and $S_C^2 \cong$ Var(CJ_k) if N is a sufficiently large integer. Taking that into account, the Var(A) and Var(S) values can be derived from equations (9,10) as follows:

$$\mathrm{Var}(A) \cong 3S_P^2 - S_C^2; \tag{11}$$

$$\mathrm{Var}(S) \cong 0.5(S_C^2 - 2\,S_P^2). \tag{12}$$

Evidently, the above design equations are applicable if ratio $R = S_P^2/S_C^2$ is in the range from 1/3 (when Var(A)=0) to 1/2 (when Var(S)=0). Otherwise one can conclude that either the statistical error of the estimation is too large or the considered theoretical model is not applicable for the particular oscillator under test. Specifically, when the ratio R is near to 1/3 or to 1/2, a good statistics is needed to separate one jitter component from another in conformity with equations (11, 12).

At first glance one might expect that experimental estimate S_A^2 is more suitable for variance Var(A) evaluation as the accumulative jitter component can be represented in this estimate more apparently. However, it should be taken into account that the A-jitter values are strongly correlated over their sequence and single estimate S_A^2 reflects only some unique realization of jitter accumulation. Correspondingly only averaged value $\bar{S}_A^2$ of estimates S_A^2 obtained in a lot of measurement cycles may be statistically reliable. This feature makes the evaluation process too time-consuming for practical application. In addition, in view of particular defining the A-jitter function with the use of time-stamps fitting, there are certain problems to express the ratio between $\bar{S}_A^2$ and Var(A) analytically in an easy-to-use form.

The above estimate (11) of Var(A) value is attributed to the measured signal period T_0 which can differ for different oscillators, containing different number of original signal periods. To simplify the comparison of different oscillators (in terms of their accumulative jitter component), let us normalize Var(A) value by the measured signal period T_0 so that $\mathrm{RMS_N}(A)$= Var(A)/T_0. In this case the jitter variance accumulated during variable period T_M,

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is equal to T_M[RMS$_N$(A)]. Thus the experimentally evaluated parameter RMS$_N$(A) can be simply applied for prediction of RMS jitter accumulated during any user-defined time interval T_M.

4.3. Some experimental data and their interpretation

To investigate applied possibilities of the considered approach to jitter analysis and to verify the validity of the jitter model considered above, we took for testing two types of high-performance crystal oscillators: a voltage-controlled oscillator (VCXO) and a clock oscillator (CXO). Each of them represents a hybrid circuit packaged into a hermetic holder that contains the oscillator and output buffer electronics. However these devices slightly differ in implementation, and therefore the manufacturer specifies better stability of the VCXO than that for the CXO.

We have performed the jitter measurement for each type of the oscillators in time-ranges of 100 and 200 ms. Periods of the measured signals were 14.084 μs (CXO) and 20.492 μs (VCXO) after preliminary division of their original frequencies. Table 1 (2-4 columns) illustrates the squared deviations S_A^2, S_P^2 and S_C^2 that were obtained by averaging of measurement data over 1000 cycles. Columns 5-6 of the table represent RMS deviations of the model parameters that are evaluated in conformity with equations (11, 12); column 7 represents the normalized estimate of A-jitter.

Table 1. Experimental data concerning clock oscillators evaluation.

1	2	3	4	5	6	7
Oscillator type	S_A^2 (ps^2)	S_P^2 (ps^2)	S_C^2 (ps^2)	RMS(A) (ps)	RMS(S) (ps)	RMS$_N$(A) (ps)
VCXO						
T_M= 100 ms	43.13	10.71	31.65	0.49	2.26	1.17 E-8
T_M= 200 ms	171.13	10.55	31.17	0.49	2.24	1.17 E-8
CXO						
T_M= 100 ms	118.66	10.80	32.08	0.57	2.29	2.31 E-8
T_M= 200 ms	307.74	10.94	32.26	0.75	2.28	3.99 E-8

On the basis of these data one can see:

- Inequality $2S_A^2 >> S_P^2$ clearly indicates that both oscillators under test are distinguished by the presence of a noticeable accumulative component of total jitter. In a certain sense this criterion seems like the Durbin-Watson statistic.
- The values RMS(A) evaluated for the VCXO actually are the same for two time-ranges in conformity with the theoretical model. However, that does not concern the CXO where the process of jitter accumulation seems more complicated.
- As the data in column 7 indicate, the VCXO is much better than the CXO in terms of jitter accumulation. Especially that concerns great time-ranges. For example, the calculated jitter for 1 sec time-range is 108 and 200 ps (RMS) for the VCXO and CXO respectively.
- In estimations of RMS(S) (column 6) the event timer error evidently dominates. Unfortunately this error cannot be precisely separated from oscillator jitter. Nevertheless, comparison of these estimates for the CXO and VCXO indicates that CXO superimposed jitter is greater (by 0.4 ps RMS approx) than that for the VCXO.
- In general, one may conclude that the VCXO offers short-term stability markedly better than that of the CXO in all respects.

5. Summary

A relatively simple unified technique of jitter measurement based on high-precision event timer application is proposed. This technique offers jitter characterization in different views, starting from the directly measured accumulated jitter that is of special interest for various applications [10]. An approach to this jitter characterization based on de-trending of the time-stamps progression is discussed. This approach allows multiple measurements of the accumulated jitter and its statistical description where the influence of long-term signal instability is significantly reduced.

An approach to statistical jitter analysis based on the simplified theoretical model of jittered clock oscillators is discussed. Such an approach provides for correct quantitative interpretation of the measurement results and experimental evaluation of the model parameters. Experimental research confirms the ability to evaluate these parameters with femtosecond precision using the event timer with a time precision of about of two picoseconds. However the model not always fully conforms to the real cases, indicating a more complicated nature of jitter accumulation for typical clock oscillators.

Generally it seems that the considered technique of jitter measurement could complement the traditional oscilloscope-based technique, offering comparable precision, simpler and cheaper solution as applied to the jitter of input signals limited in upper frequency to hundreds of MHz. Expected advantages of this technique also concern a versatile characterization of clock oscillators in a wide time-range.

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