VERSITA
GOSPODARKA SUROWCAMI MINERALNYMI
Tom 28 2012 Zeszyt 4
DOI 10.2478/v10269-012-0037-8

JACEK MUCHA*, MONIKA WASILEWSKA-BŁASZCZYK**

Variability anisotropy of mineral deposits parameters and its impact on resources estimation – a geostatistical approach

Introduction

The studies on anisotropy of deposit parameters variation (e.g., contents of desirable and undesirable components, thickness, bulk density) are rarely reported on in Polish geological periodicals. Anisotropy is understood as directional (in terms of 2D or 3D space) diversification of variability of a deposit parameter referred to its intensity or style. The importance of anisotropy modeling is demonstrated by possible applications of its results:

- more precise estimation of resources and grade of raw material or desirable component in a deposit,
- proper evaluation of interpolation accuracy of deposit parameters resulting in more credible contour maps based on such evaluation,
- optimization of sampling grid designs (boreholes, samples taken in mine workings),
- designing of optimized exploitation.

The review of various methods of anisotropy description (Chetverikov, Trembecki, geostatistical, trends and autocorrelations) has been presented by Kokesz and Mucha (1984). These authors demonstrated significant differences in anisotropy evaluations obtained with above mentioned methods and explained them as the result of different variability models (deterministic-geometric, random or mixed) used in calculations. In the last years methods of Matheron's geostatistics have found wide application in studies on Polish mineral deposits, e.g. copper-silver ores, lignite, hard coal and native sulphur (Namysłowska-Wilczyńska

---

* Professor, ** Ph.D. Eng., AGH University of Science and Technology, Kraków, Poland;
e-mail: mucha@geol.agh.edu.pl, wasilewska@agh.edu.pl

1993; Mucha et al. 2004; Kokesz 2006, 2010; Mucha, Wasilewska-Błaszczyk 2010, 2011; Bartuś 2012).

The main objective of the following paper is to evaluate the influence of geostatistically described 2D and 3D anisotropy on the results of interpolation of deposit parameters, both in the points and blocks of deposit. Examples originate from the Polish mineral deposits.

1. Outlines of geostatistical description and modeling of variability anisotropy

Geostatistical estimation of the values of deposit parameters in both the sampling points and deposit fragments (blocks), considered for both the 2D and 3D space, is based upon description of the structure of variability (see e.g., Journel, Huijbregts 1978; Deutsch, Journel 1997; Gringarten, Deutsch 2001). For such description the discrete function is applied – the empirical (sample) semivariogram which illustrates the dependence between average squared difference of given deposit parameter and average distance between sampling (measurement) sites. The "classic" empirical semivariogram for sampling data can be calculated in the two variants:

— exclusively as a function of distances between sampling sites (independently to direction of calculation) – this is an omnidirectional (or isotropic) semivariogram calculated from the formula:

$$\gamma(h) = \frac{1}{2N_h} \sum_{i=1}^{N_h} (z_{i+h} - z_i)^2$$

where:

z_i, z_{i+h} – values of analyzed parameter at sampling sites distant by h (with assumed certain distance tolerance Δh),

N_h – number of pairs of sampling sites distant by h;

— as a function of distance between sampling sites and direction of analysis – this is a directional semivariogram calculated from the formula:

$$\gamma(h,\alpha) = \frac{1}{2N_{h(\alpha)}} \sum_{i=1}^{N_h} (z_{i+h(\alpha)} - z_i)^2$$

where:

$z_{i+h(\alpha)}$, z_i – values of analyzed parameters at sampling sites distant by h in direction α (with assumed certain distance toleranceh Δh and certain angular tolerance $\Delta\alpha$ around the direction α),

$N_{h(\alpha)}$ – number of pairs of sampling sites distant by h in direction α.

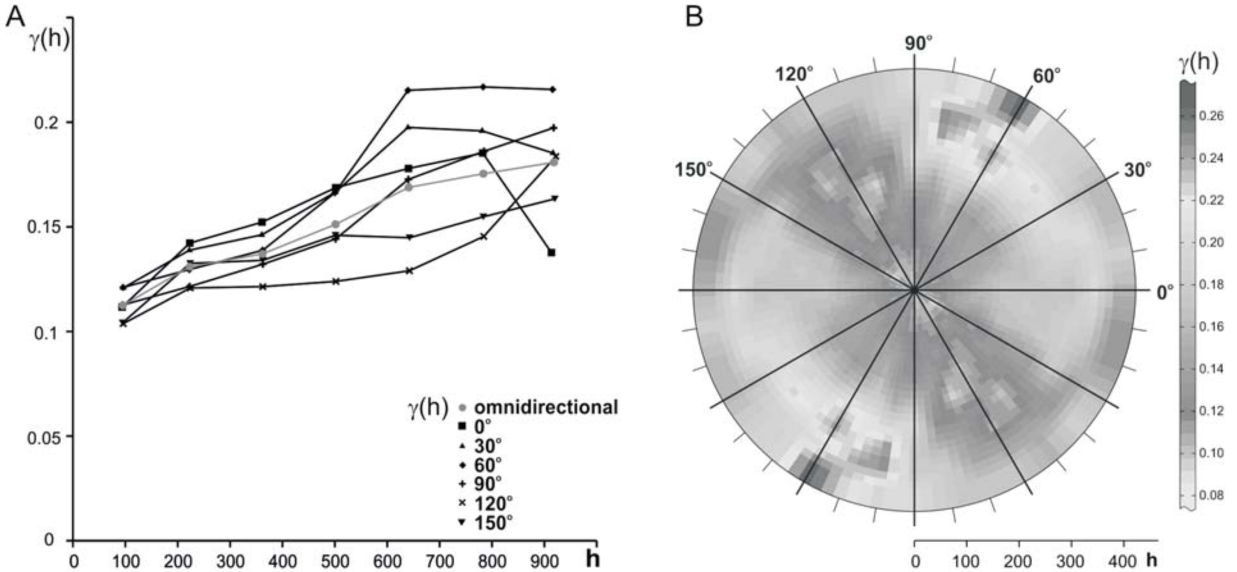


Fig. 1. Example of directional semivariograms calculated for anisotropic variabilities of parameter(A), and the example of map of directional semivariograms (B)
60° – direction of maximal variability, 120° – direction of minimal variability

Rys. 1. Przykład semiwariogramów kierunkowych w warunkach anizotropowej zmienności parametru (A) oraz przykład mapy semiwariogramów kierunkowych (B)
60° – kierunek maksymalnej zmienności, 120° – kierunek minimalnej zmienności

The direction of analysis is defined by an angle (α) which determines the deviation of data computing direction from the assumed reference direction (depending on the software used it can be e.g., 0X axis of the Cartesian coordinates or the geographic north).

The basis for description of anisotropy is the set of directional semivariograms calculated for variously oriented data computing directions. The number of directions for which credible directional semivariograms can be calculated depends on the number of basic data and increases with that number. Generally, six calculation lines deviated by 30° are required for directional semivariograms. Practical principle is accepted that calculations of semivariogram values for successive average distances between sampling sites should be based upon at least 30 pairs of parameter mesurements. Anisotropy of variability can be visualized by plotting together directional semivariograms (Fig. 1A). Their distinctly different patterns indicate the anisotropic structure of variability of given parameter. Other, more attractive visualization form of directional variability is the contour map of the values of directional semivariograms (Fig. 1B). In the case of ideal, isotropic variability the contours form concentric circles. The deviations from this “ideal pattern” are measures of the type and the power of anisotropy.

Any geostatistical estimations require the approximations of empirical semivariograms (presented as points in the diagrams – Fig. 1A) with the permitted continuous analytic functions. The set of most commonly used functions considered as geostatistical variability models is shown in Fig 2. If variability structure of analyzed parameter is complicated the composite must be applied of two or more functions for approximation of empirical semivariogram leading to the compound model, i.e., model which is a sum of two or more basic, single theoretical models.

Two basic types of anisotropy can be distinguished: simpler – geometric and more difficult for modeling – zonal (Fig. 3).

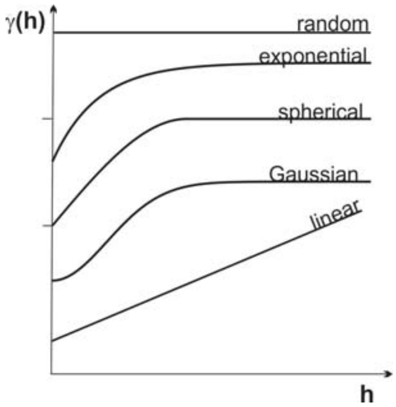


Fig. 2. Principal geostatistical variability models of a parameter
(models approximating empirical semivariograms)

Rys. 2. Wykresy podstawowych geostatystycznych modeli zmienności parametru wykorzystywane do aproksymacji semiwariogramów empirycznych (próbkowych)

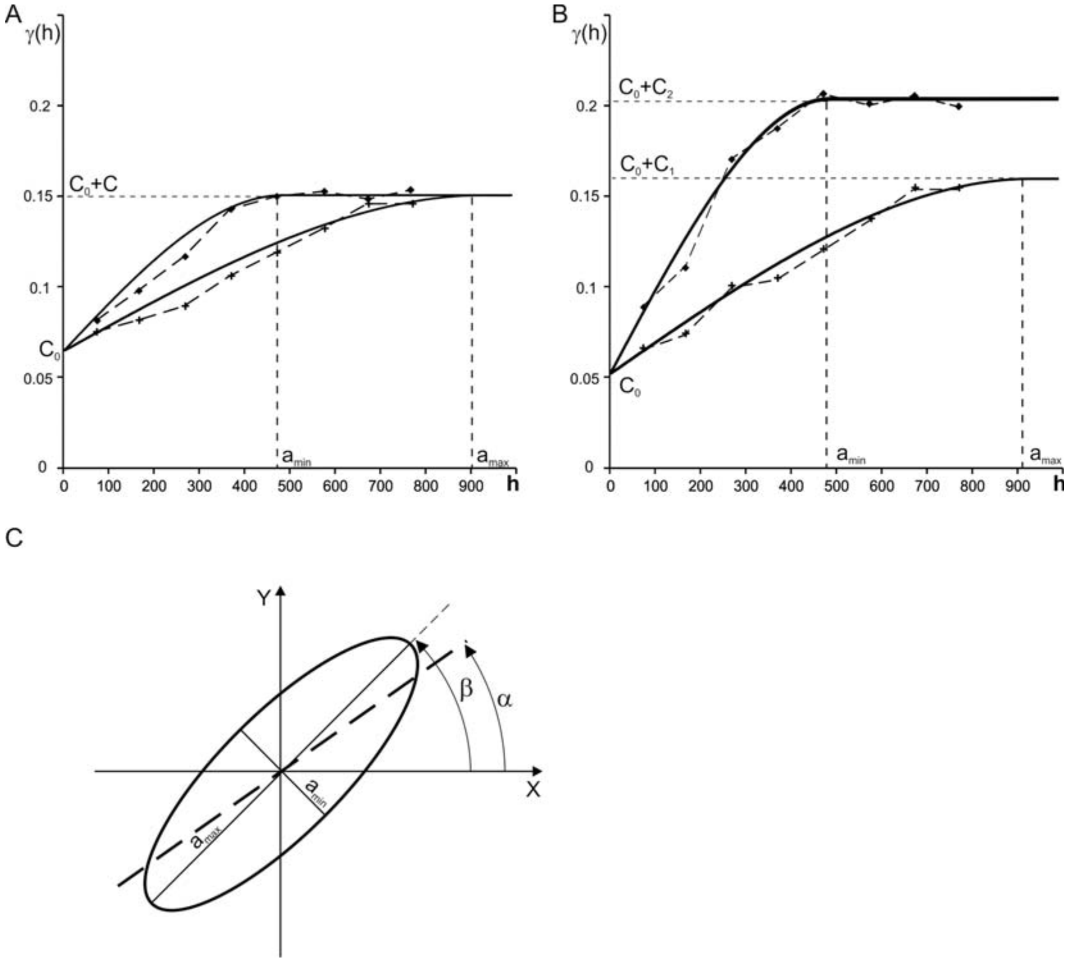


Fig. 3. Examples of variability anisotropy of given parameter: geometric (A) and zonal (B) (spherical variability model) and elliptical rose plot of semivariogram ranges in the case of geometric anisotropy (C)
$a_{\max}$ – maximal range of semivariogram (in the direction of minimal variability), $a_{\min}$ – minimal range of semivariogram (in the direction of maximal variability), α – angle between 0X axis and direction of analysis, β – angle between 0X axis and direction of longer axis of anisotropy ellipse ($a_{\max}$), C_0 – random component of variability (nugget variance), C, C_1, C_2 – non – random (spatial) components of variability

Rys. 3. Przykłady anizotropii zmienności parametru: geometrycznej (A) i zonalnej (B) w warunkach sferycznego modelu zmienności parametru oraz elipsa zasięgów semiwariogramów w warunkach anizotropii geometrycznej (C)
$a_{\max}$ – maksymalny zasięg semiwariogramu (w kierunku minimalnej zmienności), $a_{\min}$ – minimalny zasięg semiwariogramu (w kierunku maksymalnej zmienności), α – kąt między osią 0X i kierunkiem badania zmienności, β – kąt między osią 0X i orientacją dłuższej osi elipsy anizotropii ($a_{\max}$), C_0 – wariancja losowego składnika zmienności, C, C_1, C_2 – wariancje nielosowego składnika zmienności

For the most commonly used spherical model (Fig. 3A) the geometrical anisotropy appears as different ranges of semivariograms (a) for various directions of analysis at constant values of the remaining parameters of the model (C_0 and C). For the same model the zonal anisotropy is revealed by diversified values of parameters for models “a” and “C” for various directions of analysis.

The 2D geometrical anisotropy can be easily described mathematically. The function for spherical model for any direction of calculation α is given by the following formula (Webster, Oliver 2007):

$$\gamma(h,\alpha)=C_0+C\left[\frac{3}{2}\frac{h}{\Omega(\alpha)}-\frac{1}{2}\left(\frac{h}{\Omega(\alpha)}\right)^3\right]$$

for $h \le \Omega(\alpha)$

$$\gamma(h,\alpha)=C_0+C$$

for $h \ge \Omega(\alpha)$

$$\Omega(\alpha)=[a_{\max}^2 \cdot \cos^2(\alpha-\beta)+a_{\min}^2 \sin^2(\alpha-\beta)]^{1/2}$$

where:

$a_{\max}$ – maximal range of semivariogram,
$a_{\min}$ – minimal range of semivariogram (Fig. 3A, C),
α – angle between 0X axis and direction of analysis,
β – angle between 0X axis and direction of longer axis of anisotropy ellipse ($a_{\max}$) (Fig. 3C).

The power of geometric anisotropy for spherical model is usually expressed by means of the anisotropy ratio:

$$A=\frac{a_{\max}}{a_{\min}}$$

where:

$a_{\max}$ – maximal range of semivariogram,
$a_{\min}$ – minimal range of semivariogram.

However, this coefficient refers only to non-random component of variability (C) and completely ignores the random component (C_0). If the random component constitutes a significant contribution to the overall, observed variability of given parameter the high ratio value A may lead to erroneous conclusion that anisotropy is significant.

Apart from power of anisotropy, important is also spatial orientation of minimal and maximal variability directions of given parameter in relation to both the reference direction and to each other. Sometimes, the particular orientations of minimal and maximal variability can be interpreted from the genetic point of view. However, it must be emphasized that the observed style of anisotropy can change depending on the scale of observations.

For zonal anisotropy, which is more difficult for modeling, it is possible to use some mathematical transformations which lead to pseudo-geometric anisotropy equations (Isaaks, Srivastava 1989).

Obviously, the analysis of anisotropy of parameter variability in 3D space is much more difficult as it requires the introduction of additional, vertical direction of changes of parameter values.

2. Influence of geometric anisotropy on the estimation of deposit parameters values

A potential impact of geometrical anisotropy on the estimation of deposit parameters were analyzed theoretically using the simplified example shown in Fig. 4. The evaluation objective was the value of parameter in the center of 20x20 m square calculation block (point interpolation) and its average value for the whole block (block interpolation). Estimation was based upon the results of sampling obtained from four measurement sites located in the centers of block sides, i.e., equally distant from the center. For estimation the ordinary kriging procedure was applied (Journel, Huibregts 1978) assuming that the variability structure is described by spherical model.

In the ordinary kriging procedure the parameter values in both the central point and for the calculation block are estimated as weighted averages, according to formula:

$$z^{*}{}_{K} \sum_{i=1}^{n} w_i z_i$$

where:

w_i – weight of kriging ascribed to the measurement point “*i*” at: $\sum_{i=1}^{n} w_i = 1$,

z_i – value of parameter measured at point ”*i*”,

n – number of measurements points.

Values of weights were calculated from kriging equations (Journel, Huijbregts 1978), which consider geostatistical model of parameter variability, positions of measurements points in relations to point (or block) of estimation (interpolation) and to each other.

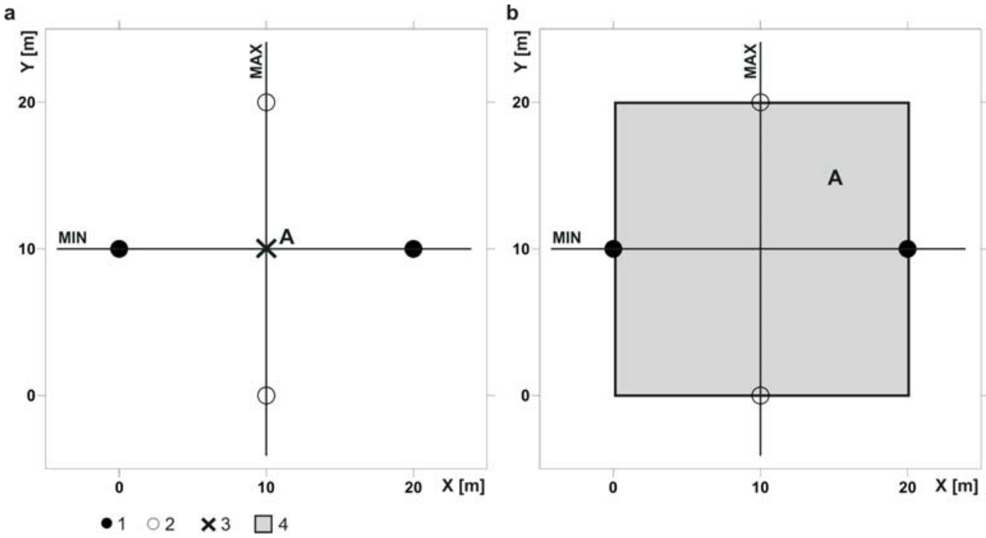


Fig. 4. Schematic presentation of parameter value estimation at point (a) and in block (b)
MIN, MAX – directions of minimal and maximal variability,
1, 2 –sampling sites located along direction respectively of minimal and maximal variability,
3, 4 – point and block of parameter value estimation

Rys. 4. Schemat oszacowania wartości parametru w punkcie (a) i bloku (b)
MIN, MAX – kierunki minimalnej i maksymalnej zmienności parametru,
1, 2 – punkty opróbowania na linii minimalnej i maksymalnej zmienności,
3, 4 – punkt i blok oszacowania wartości parametru

Calculations of weights for both the point and the block kriging were carried on for various alternatives of anisotropic variability model which vary in:

- the ranges of semivariograms of given parameter for directions of minimal ($a_{\max}$) and maximal ($a_{\min}$) variability, according to relation $a_{\max}$[m]/$a_{\min}$[m]: 20/10, 200/100, 50/10 and 500/100; the first two values correspond to moderate anisotropy (anisotropy ratio = 2) whereas two next represent extremely strong anisotropy (ratio = 5),
- the contribution of random component (C_0) to the overall variability ($C_0 + C$) equals: 0%, 25%, 50% and 75%.

For the isotropic model of variability, at the assumed pattern of four measurement sites in the calculation block values of weights ascribed to these point are identical and equal 0.25 (as their sum must equal 1). For the anisotropic model of variability the values of weights distributed along the line corresponding to minimal variability (semivariogram range $a_{\max}$) will always be higher then the values of weights located along the line of maximal variability (semivariogram range $a_{\min}$). The diversity of weights values for maximal (or minimal) variability direction at anisotropic and isotropic variability models can be a simple measure of the power of anisotropy impact on estimation of given parameter values. In the case considered here the influence of anisotropy was characterized quantitatively by means of ε_w indicator defined as the difference between value of weight for single measurement along the direction of minimal variability in the anisotropic model ($w_{\max}$) and the value of weight for isotropic model (0.25). The value ε_w of indicator was calculated from the formula:

$$\varepsilon_w = \frac{w_{\max} - 0.25}{0.25} \cdot 100\%$$

The value ε_w of indicator varies in the range of [0–100%) in which 0% corresponds to isotropic style of variability whereas the increasing power of anisotropy influencing the estimation of given parameter is characterized by values closing to 100% (Table 1).

Calculations of ε_w (%) indicator (Table 1) demonstrate that in the case of estimation of deposit parameter within calculation block anisotropy plays minor role and the relative difference of weights values for the anisotropic and isotropic models does not exceed 15%. Estimation of parameter value at the interpolation point depends not only on the anisotropy ratio and the contribution of random component of variability but also on the relationships between the ranges of directional semivariograms and the average sampling interval.

If the minimal range of directional semivariogram is smaller or similar and the maximal range is distinctly larger than the spacing of sampling sites, the relative differences of weights ε_w for directions of maximal and minimal variability are significant (>30%)

TABLE 1

Values of ε_w (%) indicator for various contribution of random component of variability (U_L) and various anisotropy ratios for point and block values of parameter estimations (Fig. 4)

TABELA 1

Wartości wskaźnika ε_w (%) dla różnych udziałów losowego składnika zmienności (U_L) i wskaźnika anizotropii przy szacowaniu wartości parametru w punkcie i bloku (rys. 4)

$U_L = \frac{C_0}{C_o + C} \cdot 100\%$		Anisotropy ratio = 2				Anisotropy ratio = 5			
		$a_{\max}$ = 20 m $a_{\min}$ = 10 m		$a_{\max}$ = 200 m $a_{\min}$ = 100 m		$a_{\max}$ = 50 m $a_{\min}$ = 10 m		$a_{\max}$ = 500 m $a_{\min}$ = 100 m	
		Point	Block	Point	Block	Point	Block	Point	Block
0%	ε_w (%)	62.4%	4.0%	68.0%	13.6%	98.0%	2.4%	95.2%	12.4%
25%	ε_w (%)	46.8%	2.8%	17.2%	3.2%	76.8%	2%	26.4%	3.6
50%	ε_w (%)	31.2%	2%	6.8%	1.2%	53.6%	1.6%	10.8%	1.6%
75%	ε_w (%)	15.6%	0.8%	2.4%	0.0%	28.4%	0.0%	4%	0.0%

U_L – contribution of random component to the overall variability of given parameter, $a_{\max}$, $a_{\min}$ – ranges of directional (spherical) semivariograms for directions of minimal and maximal variability for the contribution of random component U_L lesser than 50%. If the ranges of directional semivariograms exceed the dimensions of calculation block the influence of anisotropy will be distinct only if there exist strong regularities in parameter variability, i.e., if the contribution of random component to the overall observed variability is low ($U_L < 25\%$).

Obviously, this specific example illustrating the impact of variability anisotropy on estimations of deposit parameters does not include the immense number of possible variants of mutual relationships between the ranges of directional semivariograms, the sample spacing, the dimensions and shapes of blocks and the shares of random component in overall observed variability of given deposit parameter. These results generally confirm the experience of the authors gained from geostatistical modeling of variability of Polish mineral deposits.

3. Examples of variability anisotropy of parameters of Polish mineral deposits

Due to limited space of the following paper, only a few, selected examples are presented below in order to illustrate the general features of variability anisotropy of deposit parameters.

In the copper-silver ores from the Lubin Copper District variability of Cu content examined in horizontal plane (2D) shows relatively low anisotropy in the short scale of observation, which results from significant share of random component of variability (Fig. 5).

Different variability style is observed for Ag (and Cu) contents examined in 3D space. Here, the vertical variability is many times higher than the lateral one (Fig. 6). Therefore, any credible determination of distribution pattern of this parameter in 3D space using the kriging procedure is impossible prior to the construction of 3D model of variability anisotropy.

Another style of variability is commonly observed for thickness of hard coal seams, at least in tectonically undisturbed parts of coal deposits. For this parameter strong regularity is observed supported by poorly marked random component of variability (C_0). Sometimes, this parameter reveals strong variability anisotropy, which should be taken into account when sampling grid is designed for higher assessment categories of given mineral deposit (Fig. 7).

The contour maps of No. 207 coal seam thickness constructed from block interpolation, separately for isotropic and anisotropic models, do not reveal substantial differences (Fig. 8). It can be explained in terms of interpolation based only upon measurement sites located close to given calculation block and close to each other (about 1,500 meters) for which differences of values of directional semivariograms are relatively low.

Rather week anisotropy caused by significant contribution of random component (about 50%) was found for the sulphur accumulation (Mg/m^2) in the Osiek native sulphur deposit (Fig. 9). Therefore, contour maps of this parameter constructed for geostatistical, isotropic and anisotropic variability models do not show distinct differences (Fig. 10).

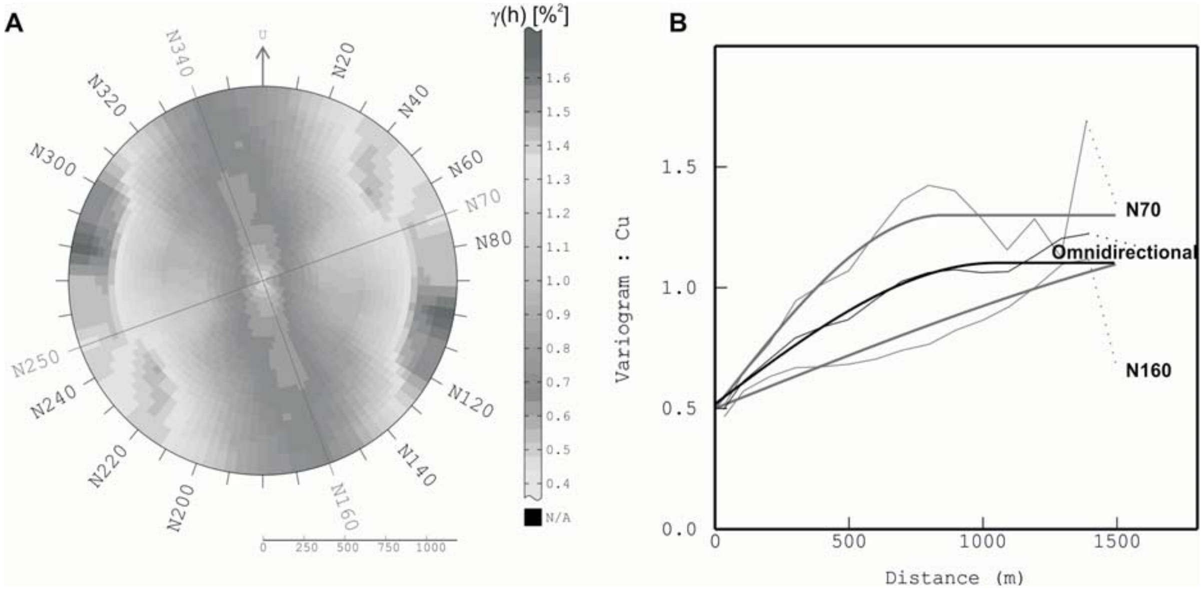


Fig. 5. Map of directional semivariograms (A) and models of 2D directional semivariograms (B) of Cu contents in Cu-Ag ores from the Lubin Copper District

Rys. 5. Mapa semiwariogramów kierunkowych (A) i modele semiwariogramów kierunkowych 2D (B) zawartości Cu w jednym ze złóż Cu-Ag LGOM

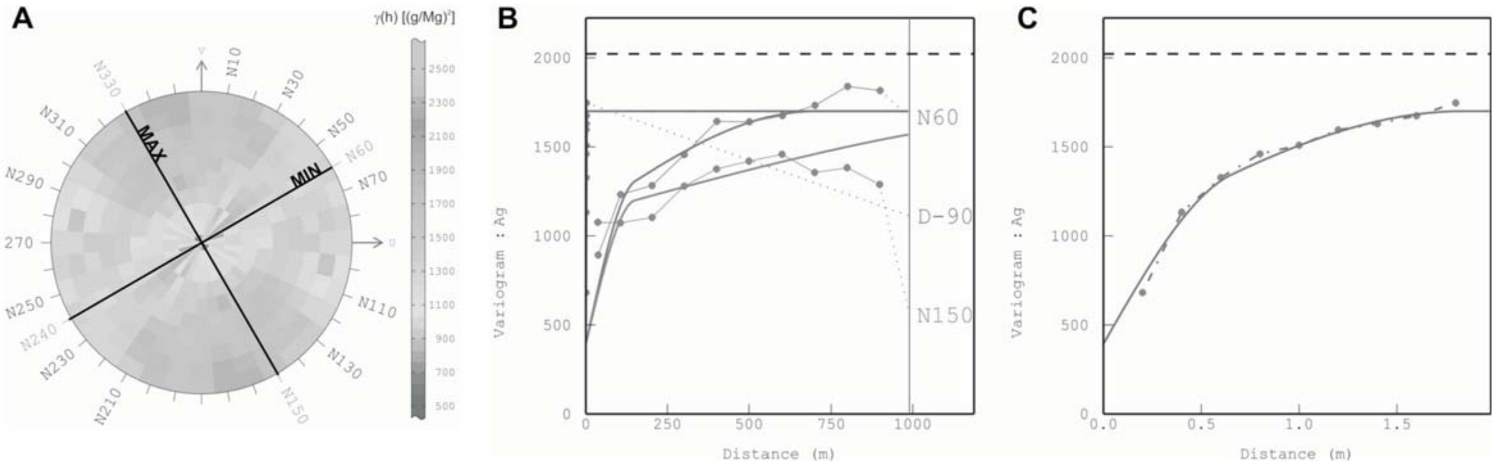


Fig. 6. Map of directional semivariograms (A) and models of 3D directional semivariograms: horizontal and vertical (B) and vertical (C) of Ag contents in Cu-Ag ores from the Lubin Copper District

Rys. 6. Mapa semiwariogramów kierunkowych (A) i modele 3D zmienności zawartości Ag w jednym ze złóż Cu-Ag LGOM (B – semiwariogramy kierunkowe poziome i pionowy, C – semiwariogram pionowy)

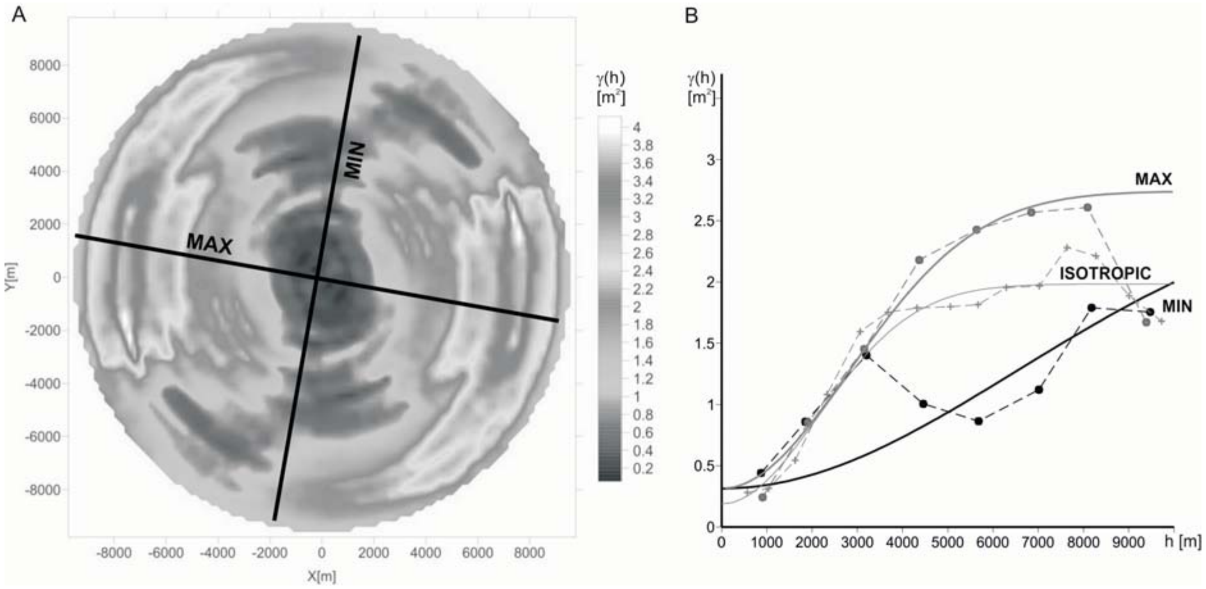


Fig. 7. Variability structure of thickness of No. 207 coal seam from an area between the Jaworzno and the Janina mining fields (Mucha, Wasilewska-Błaszczyk 2011) A – map of directional semivariograms, B – semivariograms: isotropic and for directions of minimal (MIN) and maximal (MAX) variability

Rys. 7. Struktura zmienności miąższości pokładu węgla kamiennego 207 między OG Jaworzno i OG Janina (Mucha, Wasilewska-Błaszczyk 2011) A – mapa semiwariogramów kierunkowych, B – semiwariogram uśredniony oraz semiwariogramy dla kierunku maksymalnej (MAX) i minimalnej (MIN) zmienności

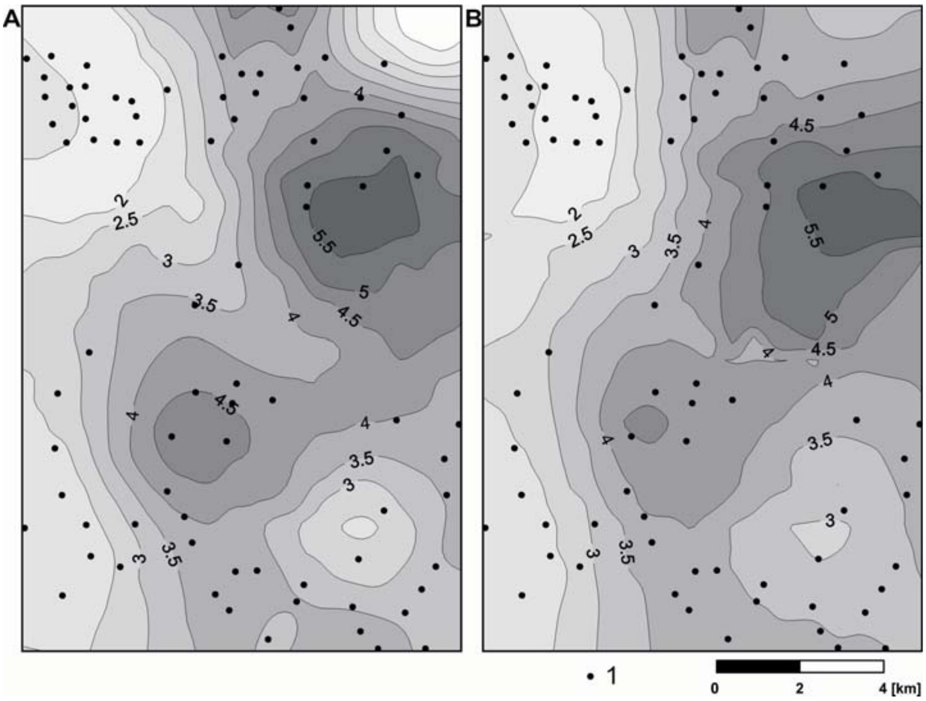


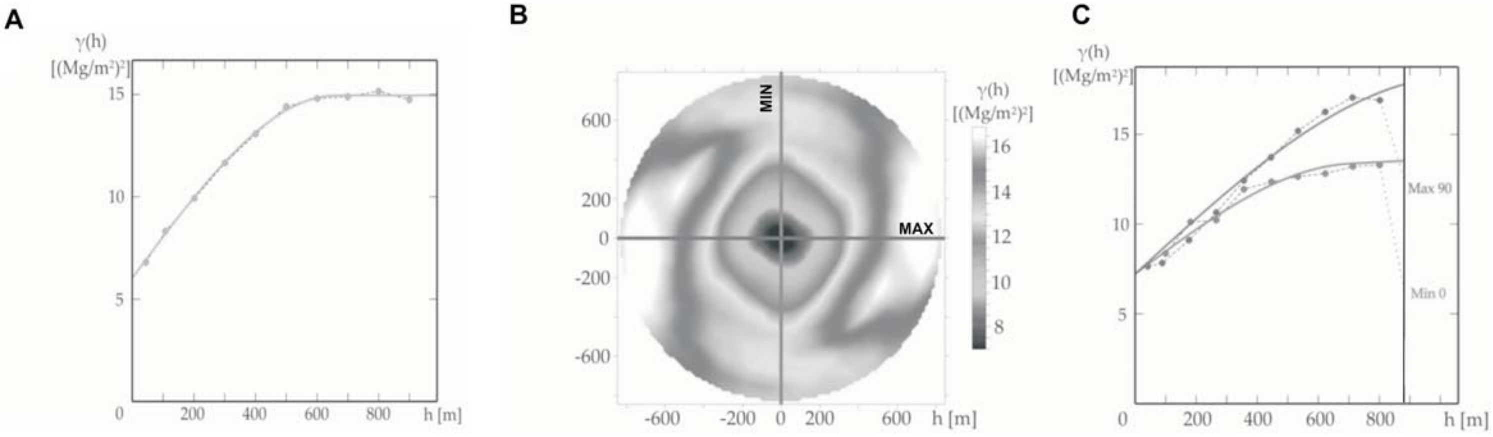
Fig. 8. Contour maps of No. 207 coal seam thickness constructed with the kriging method for isotropic (A) and anisotropic (B) semivariograms models (area between the Jaworzno and the Janina mining fields, interpolation for 0.5 × 0.5 km blocks) (after Mucha, Wasilewska-Błaszczyk 2011)
1 – borehole

Rys. 8. Mapy izoliniowe miąższości pokładu 207 między OG Jaworzno i OG Janina sporządzone metodą krigingu zwyczajnego dla modeli semiwariogramów izotropowych (A) i anizotropowych (B) (interpolacja blokowa: 0,5 × 0,5 km) (Mucha, Wasilewska-Błaszczyk 2011)
1 – otwór wiertniczy

Ambiguous results of variability anisotropy examinations were obtained for parameters describing the quality of lignite deposits. For example, in the Bełchatów deposit, variability anisotropy of sulphur and ash contents is strong (see Mucha et al. 2004) whereas in the undeveloped Gubin deposit is week with distinct random component of variability (Fig. 11).

An example of compound anisotropy of variability was found for amber accumulation (g/m^2) in the Wiślinka I amber deposit (Fig. 12). If large scale of observation is applied (corresponding to distances between sampling sites about 100 meters) strong anisotropy of variability was observed. At smaller scale (distances about 20–50 meters) anisotropy is weaker and directions of maximal and minimal variability of accumulation are swapped in relation to those for larger scale of observations.

Weak anisotropy of variability at dominating share of random component was observed for analysis of variability structure of accumulation (kg/m^2) calculated for polymetallic nodules from the Pacific Ocean floor, economic zone managed by the Interoceanmetal Joint Organization (IOM) (Fig. 13). However, it must be emphasized that in this case the results of



Rys. 9. Variability structure of sulphur accumulation for the Osiek native sulphur deposit (after Kaczmarczyk et al. 2012)
model of omnidirectional semivariograms (A), map of directional semivariograms (B), semivariograms for directions of maximal and minimal variability (C)

Rys. 9. Struktura zmienności zasobności siarki w złożu Osiek (Kaczmarczyk i in. 2012)
model semiwariogramu uśrednionego (A), mapa semiwariogramów kierunkowych (B) i semiwariogramy dla kierunku maksymalnej i minimalnej zmienności (C)

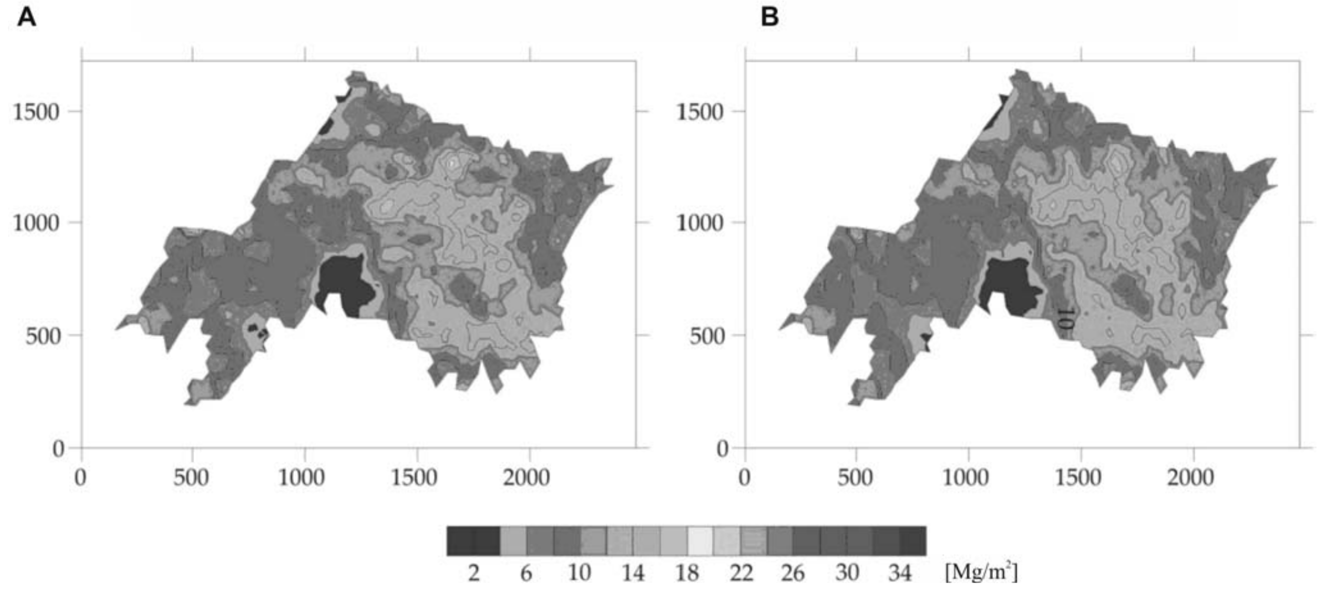


Fig. 10. Contour maps of sulphur accumulation index for isotropic (A) and anisotropic (B) variability models – example from the Osiek native sulphur deposit (after Kaczmarczyk et al. 2012)

Rys. 10. Mapy izoliniowe zasobności siarki sporządzone dla izotropowego (A) i anizotropowego modelu zmienności – złoże siarki rodzimej Osiek (Kaczmarczyk i in. 2012)

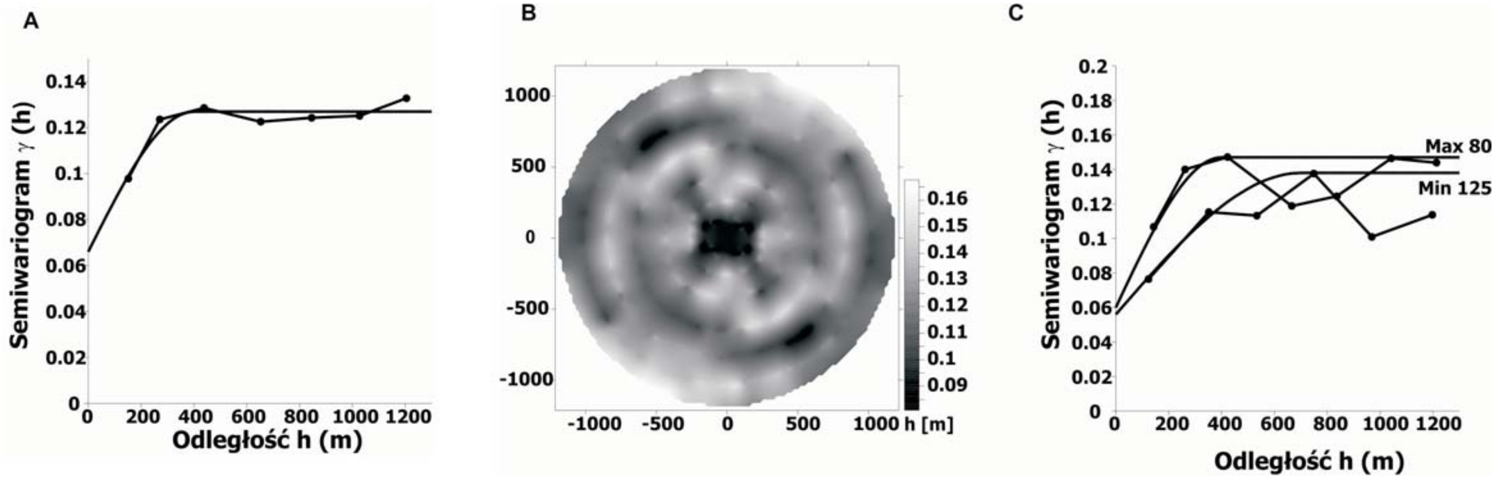


Fig. 11 Variability structure of sulphur content in the Seam I of the Gubin lignite deposit
model of omnidirectional semivariogram (A), map of directional semivariograms (B), directional semivariograms and their models for nonorthogonal directions of maximal and minimal variability (C)

Rys. 11. Struktura zmienności zawartość siarki w pokładzie I złoża węgla brunatnego Gubin
model semiwariogramu uśrednionego (A), mapa semiwariogramów kierunkowych (B), semiwariogramy kierunkowe i ich modele dla nieortogonalnych kierunków maksymalnej i minimalnej zmienności (C)

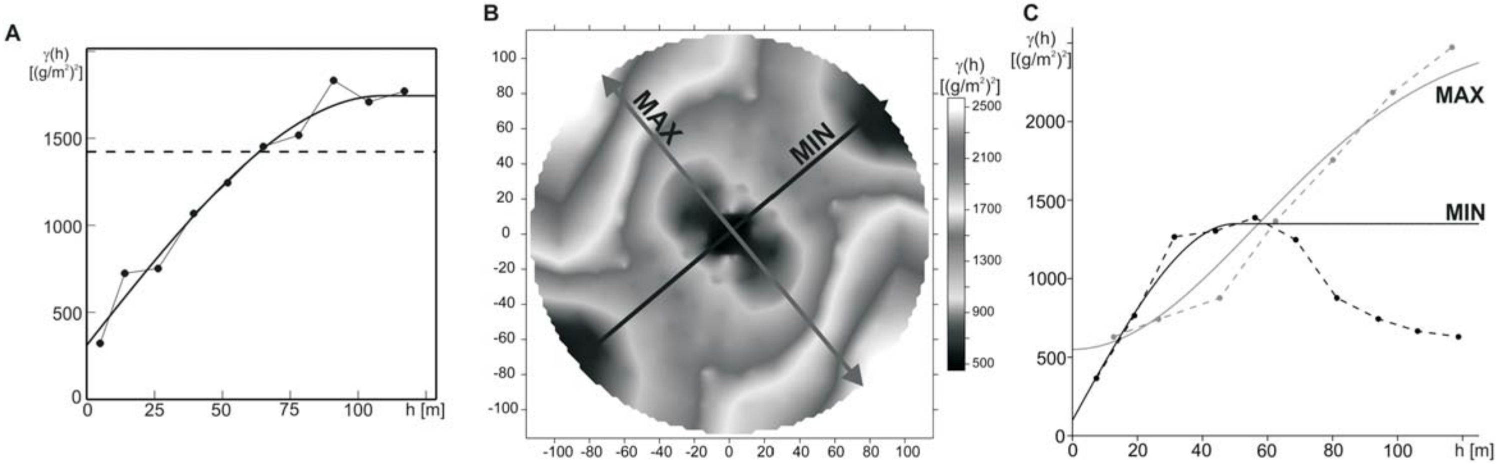


Fig. 12. Variability structure of amber accumulation index in the Wiślinka I deposit near Gdańsk (after Mucha, Wasilewska-Błaszczyk 2011)
A – model of omnidirectional semivariogram, B – map of directional semivariograms, C – semivariograms of amber accumulation index in directions of minimal (MIN) and maximal (MAX) variabilities

Rys. 12. Struktura zmienności zasobności bursztynu w złożu Wiślinka I koło Gdańska (Mucha, Wasilewska-Błaszczyk 2011)
A – model semiwariogramu uśrednionego, B – mapa semiwariogramów kierunkowych, C – semiwariogramy zasobności bursztynu w kierunku maksymalnej (MAX) i minimalnej (MIN) zmienności

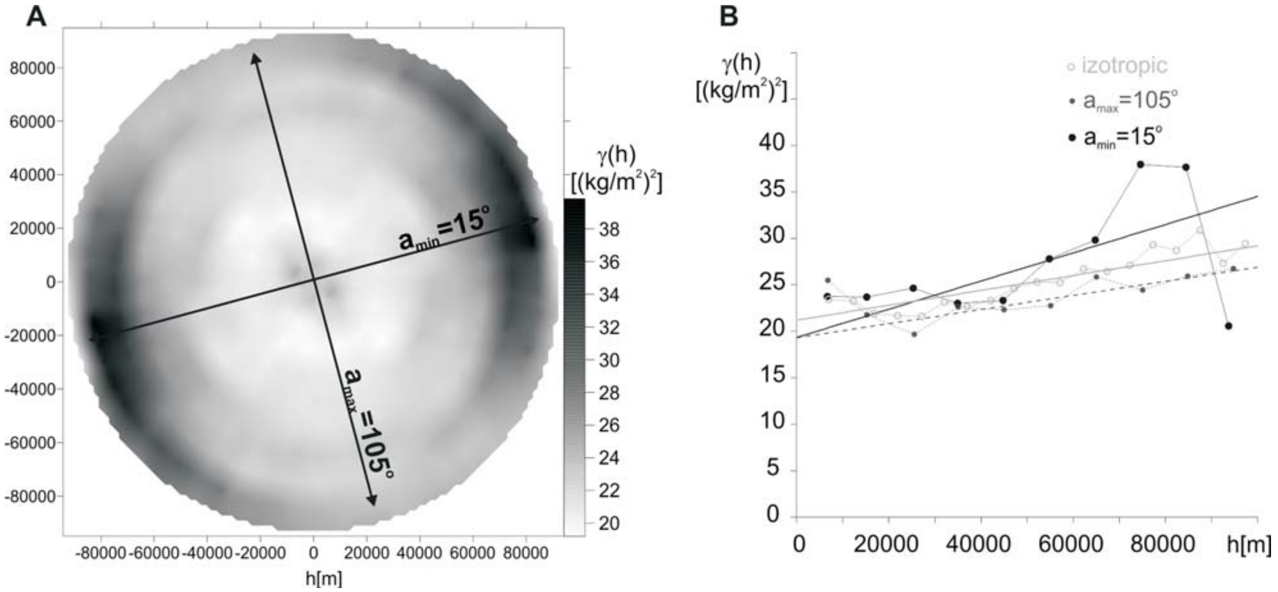


Fig. 13. Variability structure of accumulation index for polymetallic nodules from the IOM – Clarion Clipperton economic zone in the Pacific Ocean (after Mucha et al. 2011)

A – map of directional semivariograms, B – omnidirectional (isotropic) and directional semivariograms

Rys. 13. Struktura zmienności zasobności konkrecji polimetalicznych IOM – strefa Clarion Clipperton na Pacyfiku (Mucha i in. 2011)

A – mapa semiwariogramów kierunkowych, B – semiwariogram uśredniony i semiwariogramy kierunkowe anisotropy examinations refer to large scale of observations (tens of kilometers) applied for the preliminary stage of exploration when the average spacing of sampling sites is large and varies from 11 to 15 kilometers.

The brief review of variability structure description reveals a high diversity of possible styles of directional variability determined for deposit parameters from various Polish mineral deposits. Such diversity implies that in each case an individual approach is necessary in modeling of variability anisotropy. It is suggested, however, that in both the point and block interpolations the application of anisotropic models (based on directional semivariograms) instead of simpler, isotropic models (based on omnidirectional semivariograms) will not increase significantly the accuracy of interpolations, at least in most cases. Such conclusion is supported by generally high contributions of random component to overall variability and relatively low differences of values of directional semivariograms for short spacing between observations taken into account in the kriging interpolation algorithm.

4. Summary and conclusions

1. Geostatistical analysis and modeling of variability anisotropy of deposit parameters are not easy, particularly in the 3D space, if a compound form of anisotropy occurs. Moreover, credible modeling of anisotropy may be impossible at the initial stages of exploration and assessment of the deposits when number of data is insufficient. According to some authors (Webster, Oliver 2007), calculation of fully credible, omnidirectional semivariograms in 2D space requires at least 100 sampling (or measurement) sites but for calculation of directional semivariograms at least 250 values of analyzed deposit parameter must be ensured.
2. When geometric anisotropy is evaluated, not only the ranges of semivariograms referred to average spacing of measurement sites must be taken into account but also the contributions of random component to the overall variability must be examined. If the presence of this random component is ignored, it may lead the geologist to false conclusions. Even the disclosure of strong differences of directional semivariogams ranges but combined with the high random variability of given deposit parameter enables us to neglect the role of anisotropy in estimations of parameters values.
3. Generally, the estimation of parameter values in points (point interpolation) based upon the anisotropic model is effective if the random component shares less then 50% of overall observed variability of given parameter.
4. Variability anisotropy of deposit parameters in most Polish mineral deposits analyzed for the 2D space can be neglected due to high share of random component. Hence, the proper description of variability is ensured by isotropic models based upon omnidirectional semivariograms, which are much simpler to define.
5. Variability analysis of usable deposit components in the 3D space commonly confirm strong anisotropy due to higher vertical variability of values in comparison with the

lateral one. Such characteristics of variability justifies the construction of 3D anisotropic models of variability for metals content in ore deposits. The Lubin copper ore deposit is the good example (Mucha, Wasilewska-Błaszczyk 2010, 2011).

The research project was financed from the AGH University of Science and Technology grant No. 11.11.140.562

REFERENCES

Bartuś T., 2012 – Anizotropia zmienności głównych parametrów jakości węgla brunatnego w polu Bełchatów. Gosp. Sur. Min., IGSMiE PAN, Kraków, t. 28, z. 2, s. 5–29.

Deutsch C.V., Journel A.G., 1997 – GSLIB: Geostatistical software library and user's guide, 2nd Edition: Oxford University Press, New York, 369 p.

Gringarten E., Deutsch C.V., 2001 – Variogram Interpretation and Modeling. Math. Geology, vol. 33, no. 4, p. 507–534.

Isaaks E.H., Srivastava R.M., 1989 – An Introduction to Applied Geostatistics. Oxford University Press New York, 561 p.

Journel A.G., Huijbregts C. J., 1978 – Mining geostatistics: Academic Press, London, 600 p.

Kaczmarczyk et al. 2012 – Kaczmarczyk M., Nieckula M., Mucha J., Wasilewska-Błaszczyk M., 2012 – Praktyczne konsekwencje geostatystycznego badania struktury zmienności parametrów złoża węgla brunatnego Gubin i siarki Osiek. Zesz. Nauk. IGSMiE PAN nr 83, s. 51–68.

Kokesz Z., Mucha J., 1984 – Anizotropia parametrów złożowych – cele i metody badania. Tech. Poszuk. Geol. nr 2, s. 40–49.

Kokesz Z., 2006 – Application of linear geostatistics to evaluation of Polish mineral deposits. Gosp. Sur. Miner., t. 22, z. 2, p. 53–65.

Kokesz Z., 2010 – Sporządzanie map izoliniowych procedurą krigingu zwyczajnego – korzyści i ograniczenia. Zesz. Nauk. IGSMiE PAN, nr 79, s. 363–382.

Mucha et al. 2004 – Mucha J., Słomka T., Mastej W., BartuśT., Jończyk M., Frankowski R., 2004 – Modelowanie zmienności i dokładność oszacowania jakości węgla brunatnego w złożu Bełchatów (pole Bełchatów). Mat. Symp. Warsztaty 2004 z cyklu „Zagrożenia naturalne w górnictwie". Wyd. IGSMiE PAN, Kraków, s. 221–232.

Mucha J., Wasilewska-Błaszczyk M., 2010 – Prognoza jakości urobku metodami geostatystyki 3D – perspektywy i ograniczenia. Gosp. Sur. Min., t. 26, z. 2, s. 57–67.

Mucha J., Wasilewska-Błaszczyk M., 2011 – Praktyczne doświadczenia geostatystycznego modelowania i dokumentowania polskich złóż – przegląd wybranych zastosowań [W]: Geomatyka górnicza – praktyczne zastosowania (red. Dyczko A. i Krawczyk A.). Wyd. Fundacja dla AGH, Kraków, s. 129–151.

Mucha et al. 2011 – Mucha J., Kotliński R., Wasilewska-Błaszczyk M., 2011 – Zasoby konkrecji – procedury i wymagania dla ich szacowania na przykładzie obszaru IOM w polu Clarion – Clipperton. Górnictwo i Geoinżynieria, r. 35, z. 4/1, Wyd. AGH, s. 291–302.

Mucha J., Wasilewska-Błaszczyk M., 2012 – Modelowanie złóż kopalin stałych metodami 2D i 3D. Mat. konf. (CD) Szkoła Górnictwa Odkrywkowego, KGO, WGiG AGH 27–28 marca 2012, Kraków.

Namysłowska-Wilczyńska B., 1993 – Zmienność złóż rud miedzi na monoklinie przedsudeckiej w świetle badań geostatystycznych. Prace Nauk. Inst. Geotechniki i Hydrotechniki Polit. Wrocł., S. Monografie, nr 21, Wrocław, s. 205.

Webster R., Oliver M. A., 2007 – Geostatistics for Environmental Scientists. John Wiley &Sons Ltd.

ANIZOTROPIA ZMIENNOŚCI PARAMETRÓW I JEJ WPŁYW NA SZACOWANIE ZASOBÓW W UJĘCIU GEOSTATYSTYCZNYM

Słowa kluczowe

Parametr złożowy, geostatystyka, semiwariogram, anizotropia, kriging zwyczajny

Streszczenie

Anizotropia zmienności parametrów złożowych tylko sporadycznie bywa przedmiotem zainteresowania dokumentatorów polskich złóż . Jej opis i modelowanie matematyczne ma jednak sens i uzasadnienie, gdy jest ona silnie wyrażona, ponieważ tylko wówczas może wpływać zauważalnie na dokładność rozwiązania wielu zadań z zakresu geologii górniczej i górnictwa, takich jak: szacowanie zasobów i jakości kopaliny, interpolacja wartości parametrów złożowych i kreślenie ich map izoliniowych, projektowanie eksploatacji uśredniającej czy też projektowanie zagęszczonej sieci punktów rozpoznania (opróbowania) złoża. W geostatystyce opisu anizotropii dokonuje się za pomocą semiwariogramów kierunkowych wyrażających średnie zróżnicowanie wartości badanego parametru złożowego dla różnych kierunków badań w zależności od odległości między punktami opróbowań. Graficznie anizotropię ilustruje się najczęściej w wygodnej dla interpretacji formie mapy semiwariogramów kierunkowych oraz matematycznie za pomocą funkcji analitycznych ciągłych opisujących modele anizotropowe.

W artykule przedstawiono wyniki geostatystycznego opisu różnego rodzaju anizotropii zmienności parametrów złożowych na wybranych przykładach polskich złóż kopalin stałych. Teoretycznie, na przykładzie sferycznego modelu zmienności zilustrowano wpływ anizotropii na wyniki szacowania wartości parametrów złożowych w punktach i blokach złoża. Stwierdzono, że efektywność uwzględnienia anizotropii w szacowaniu parametrów złożowych jest uzależniona od wzajemnych relacji trzech elementów: siły kierunkowego zróżnicowania zmienności parametru, udziału losowego składnika zmienności w całkowitej, obserwowanej zmienności parametru oraz zasięgu autokorelacji wartości parametru odniesionego do średniego rozstawu punktów opróbowań. Wykazano, że anizotropia ma znacznie silniejszy wpływ na oszacowanie wartości parametrów w punktach złoża niż na oszacowanie jego średnich wartości w blokach złoża. Stwierdzono, że anizotropia nie ma istotnego znaczenia w przypadku dominacji losowego składnika zmienności w obserwowanej zmienności analizowanego parametru, co upoważnia do stosowania w oszacowaniach geostatystycznych prostszego, izotropowego modelu zmienności parametru.

VARIABILITY ANISOTROPY OF MINERAL DEPOSITS PARAMETERS AND ITS IMPACT ON RESOURCES ESTIMATION – A GEOSTATISTICAL APPROACH

Key words

Deposit parameters, geostatistics, semivariogram, anisotropy, ordinary kriging

Abstract

Anisotropy of variations of Polish mineral deposit parameters is rarely the subject of interest of geologists who carry on the assessment projects . However, if the anisotropy is strong its description and mathematical modeling are rational and justified as it may affect the accuracy of many calculations suitably for mining geology and mining engineering, e.g. estimation of resources and grade of particular raw-material, interpolation of deposit parameters values and construction of their contour maps, designing of optimum grade mining operations or densification of sampling grid. In geostatistics anisotropy is described with directional semivariograms which represent average variability of values of particular deposit parameter in various directions, depending on the distance between sampling sites. Convenient graphic presentation of anisotropy is map of directional semivariograms and good mathematical presentation are functions describing the anisotropy models.

The paper presents the results of geostatistical descriptions of various anisotropy types in selected examples of Polish mineral deposits. Taking into account the spherical variability model, the influence of anisotropy on the results of deposit parameters estimations has been theorized for both the interpolation point and calculation block (area). It was found that anisotropy is effective for parameters estimation if three mutually interrelated factors are considered: power of directional diversification of parameters variation, contribution of random component to total, observed variation of parameters and the range of semivariograms (autocorrelation) of parameter referred to the average sampling grid density.

The results demonstrate that anisotropy influences much more the estimations of parameters value in interpolation points than those of average values of parameters calculated for particular parts of deposit (calculation blocks). Moreover, anisotropy is unimportant when the random component of variability dominates the overall variability of analyzed parameter. Therefore, the simpler, isotropic variability model can be applied to geostatistical estimations of deposit parameters.