

## The accuracy of loss prediction in magnetic materials\*

JAN SZCZYGLÓWSKI<sup>1</sup>, PAWEŁ KOPCIUSZEWSKI<sup>1</sup>, KRZYSZTOF CHWASTEK<sup>1</sup>,  
MARIUSZ NAJGEBAUER<sup>1</sup>, WIESŁAW WILCZYŃSKI<sup>2</sup>

<sup>1</sup>*Częstochowa University of Technology  
Armii Krajowej 17, 42-200 Częstochowa, Poland  
e-mail: jszczyg@el.pcz.czyst.pl; pkop@imi.pcz.pl  
e-mail: {krzych/najgebauer}@el.pcz.czyst.pl*

<sup>2</sup>*Institute of Electrical Engineering  
Pożaryskiego 28, 04-703 Warszawa, Poland  
e-mail: w.wilczynski@iel.waw.pl*

(Received: 02.07.2010, revised: 29.11.2010)

**Abstract:** The paper presents a formula useful for prediction of loss density in soft magnetic materials, which takes into account multi-scale energy dissipation. A universal phenomenological  $P(B_m, f)$  relationship is used for loss prediction in chosen soft magnetic materials. A bootstrap method is used to generate additional data points, what makes it possible to increase the prediction accuracy. A substantial accuracy improvement for estimated model parameters is obtained in the case, when additional data points are taken into account. The proposed description could be useful both for device designers and researchers involved in computational electromagnetism.

**Key words:** soft magnetic materials, loss prediction, bootstrap technique

### 1. Introduction

The fundamental parameter used by technologists in the processes aimed at tailoring properties of produced magnetic material as well as in design and work analysis of magnetic circuits used in electric devices is loss density. Its value is usually given in the catalogues supplied by the producers for given values of flux density and frequency. According to EN 10106 standard, the fundamental quantity examined for non-oriented steel sheets is total loss density measured at 1.5 [T]. Very often loss density at 1.0 [T] is also supplied. For grain-oriented steel sheets, the fundamental quantity being examined is total loss measured at 1.7 [T] (EN 10107 standard), but some designers also request information on total loss density at 1.5 [T]. Some-

---

\*This is extended version of a paper which was presented at the 21st *Symposium on Electromagnetic Phenomena in Nonlinear Circuits*, Essen-Dortmund, 29.06-02.07, 2010.

times, in particular for materials working at increased frequencies, the dependencies of loss density on maximum flux density and excitation frequency are also given. These take the form of power laws with exponents specific for given energy dissipation mechanisms. Bertotti assumes that eddy currents, generated in different time and spatial scales within the magnetized material, are the fundamental cause for energy dissipation [1, 2].

The distinction between different eddy current scales, i.e. a macro-scale, covering the whole bulk material and a micro-scale covering the area of moving domain walls, introduced by Bertotti's theory, has lead to the following relationship:

$$P = c_1 f B_m^\beta + c_2 \sigma f^2 B_m^2 + 8\sqrt{(\sigma G S V_0)} f^{1.5} B_m^{1.5}, \quad (1)$$

where  $\sigma$  is conductivity,  $G$  is a constant equal to 0.1356,  $S$  is sample cross-section, whereas  $V_0$  is a parameter dependent on flux density. The relationship is valid only for such a frequency range, when the skin effect may be neglected [1]. For example the upper limiting frequency for steel gauge 0.35 [mm] is 225 [Hz], whereas for steel gauge 0.5 [mm] is only 121 [Hz] [3]. Assumption of multi-scale energy dissipation imposes a limitation onto the aforementioned Equation (1), namely  $\beta = 1$  [4]. For most results presented in literature the condition is not fulfilled. In [5] a universal relationship, which takes into account the multi-scale energy dissipation within a magnetic material, has been proposed. The relationship takes the form:

$$P = c_1 f B_m^{(\beta-\alpha)} + c_2 f^2 B_m^{(\beta-2\alpha)}. \quad (2)$$

In the aforementioned paper it has also been proven, that the Equation (2) allowed for description of loss phenomena in soft magnetic materials of different structure, e.g. grain-oriented (GO) and non-oriented (NO) electrical steels, Ni-Fe alloys, amorphous and nanocrystalline ribbons for frequencies up to 400 [Hz]. The existence of a universal phenomenological dependence between the exponents  $\alpha$  and  $\beta$  in the form  $\beta = 1.35\alpha + 1.75$  was proven.

In the presented paper a method aimed at improving the accuracy of Equation (2) to measurement data referring to GO and NO 3% wt. Si steels is presented. The measurement points were obtained for flux densities from the range  $\langle 0.1; 1.8 \rangle$  [T] and frequencies from the range  $\langle 1; 500 \rangle$  [Hz]. The proposed algorithm is based on a statistical supplementation of the measurement data set, which is used for fitting. The presented paper is an extension of previous work [6], where the possibility to limit loss prediction errors by a random supplementation of measurement data, but without the imposed condition  $\beta = 1.35\alpha + 1.75$ , has been proven.

## 2. Measurements

The verification was carried out for a NO steel grade V350-50A (0.5 [mm] thick) and a GO steel grade 111-35-N5 (0.35 [mm] thick). Loss density measurements were made on  $500 \times 500$  [mm] square single sheets using a Single Sheet Tester device and the measurement stand MAG-RJJ-2.0. The measurement setup provided sine flux wave in the tested samples. A detailed description of the setup is given in [7]. The extended type B uncertainty did not exceed 1.5%. The total extended uncertainty of the measurement setup, which took into ac-

count the spread of material properties due to manufacturing conditions and the errors introduced by the measurement setup, did not exceed 10%.

### 3. Method

The Equation (2) used for the description of energy loss in magnetic materials is a nonlinear function of two variables  $f$  and  $B_m$  with four parameters  $\alpha$ ,  $\beta$ ,  $c_1$  and  $c_2$ . The Equation (2) was validated using raw measurement points. The number of measurement points was equal to 144 for the GO steel and 120 for the NO steel. Taking into account a relatively small number of data points and their nonuniformity it could happen, that the Equation (2) might be fitted only to a specific chosen range of variables  $f$  and  $B_m$ .

In statistics there are two possible approaches, which make it possible to avoid this effect. The first one is based on taking different weights of data points into account, the other one is based on resampling. In fact, both methods are equivalent to some extent. In the paper the resampling method was chosen in order to obtain a representative experimental data set. More than 100.000 additional points for the variable  $f$  using Monte-Carlo methods were randomly generated.

It can be stated that generation of additional data points makes it possible to get rid of:

- extremely high values of errors, which are caused by too few experimental data points,
- the influence of data points that fall behind the overall trend,
- model domination with values of data point concentrated around specific ranges of flux density and frequency.

In recent years, the estimation procedures based on independent replications of the initial population obtained from Monte Carlo method are the subject of intensive research [8, 9].

Generation of additional points for a fixed value of flux density was carried out according to the following schema:

- a. the value of flux density  $B_m$  was fixed,
- b. the whole range of variability of  $f$  and  $P$  was divided into exclusive sets,
- c. for the first specified set the additional data points  $f$  and  $P$  were randomly generated according to a two-dimensional normal distribution, whose expected value and covariance matrix were estimated on the basis of all values of the vector  $(f, P)$  in the neighbourhood of the specified set. During generation the points lying closer to the specified range were assigned higher weights when calculating the distribution parameters for generation,
- d. generation of points  $(f, P)$  for a subsequent set was carried out in the same manner as described in paragraph (c), but at every subsequent step the data points generated previously were additionally taken into account,
- e. if all the sets of the  $(f, P)$  plane were swept, the value of flux density was incremented and the whole procedure was restarted from the paragraph (a).

The procedure is illustrated in Figures 1 and 2.

In a similar manner, for a fixed value of frequency  $f$  the points  $(B_m, P)$  were generated. In this way the second data set was created. Then both sets were combined into one. The combined set was used for estimation of values of parameters and coefficients in the Equation (2), taking into account the condition  $\beta = 1.35\alpha + 1.75$ .

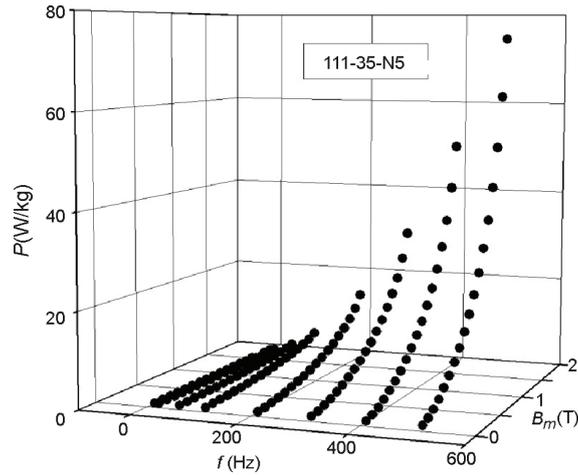


Fig. 1. Distribution of measurement points  $(P, B_m, f)$  for grain-oriented steel 111-35-N5 [6]

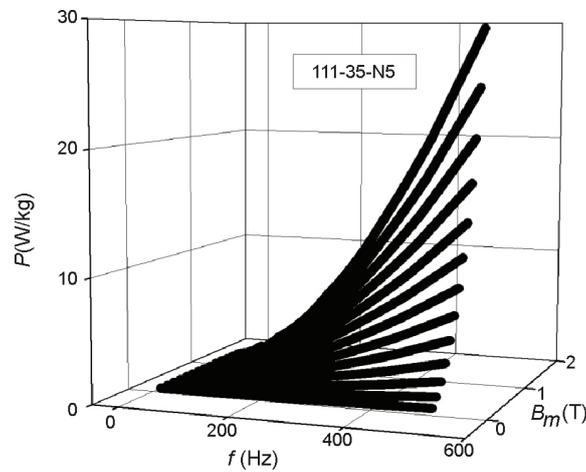


Fig. 2. Distribution of measurement points supplemented with statistically generated additional data points for grain-oriented steel 111-35-N5 [6]

#### 4. Results

For estimation of parameters  $\alpha$ ,  $c_1$  and  $c_2$ , the method of nonlinear regression, based on minimization of mean square error using the Gauss-Newton algorithm, was used. The estima-

tion quality was evaluated using the coefficient of variation of these parameters. It states, how large is the standard deviation referred to the mean value. If the value of the coefficient is big, it means, that the estimator of the considered model parameter is assessed with a larger uncertainty. The results are given in Table 1.

Table 1. The estimation results for the parameters  $\alpha$ ,  $c_1$  and  $c_2$  in the Equation (2) applied to a GO steel grade 111-35-N5 and a NO steel grade V350-50A

Type	Parameter	Estimator value	Estimator deviation	Lower bound of 95% confidence interval	Upper bound of 95% confidence interval	$V$ [%]
GO	$\alpha$	-2.27280	0.08760	-2.44610	-2.09950	3.85
	$c_1$	0.01160	0.00058	0.01040	0.01270	5.03
	$c_2$	0.00004	0.00000	0.00003	0.00004	5.58
GO <sup>x</sup>	$\alpha$	-0.63790	0.00118	-0.64020	-0.63560	0.18
	$c_1$	0.00378	0.00003	0.00372	0.00384	0.82
	$c_2$	0.00009	0.00000	0.00009	0.00009	0.33
NO	$\alpha$	-1.55910	0.03340	-1.62530	-1.49280	2.14
	$c_1$	0.02440	0.00077	0.02280	0.02590	3.18
	$c_2$	0.00010	0.00000	0.00013	0.00015	4.01
NO <sup>x</sup>	$\alpha$	-0.22870	0.00231	-0.23320	-0.22420	1.01
	$c_1$	0.01810	0.00019	0.01770	0.01850	1.08
	$c_2$	0.00015	0.00000	0.00014	0.00015	1.35

$V$  – estimator variability coefficient [%],  $x$  – estimation results for the case, when additional loss density data points were generated

Comparing the values of estimated parameters and exponents for raw measurement data points and for both experimental and additionally generated data points it can be stated, that a substantial accuracy improvement for estimated parameters was obtained in the latter case. For the grain-oriented steel, the accuracy improvement is better of one order.

In order to assess the fitting error for the Equation (2) applied to measurement data points in the considered electrical steels, the average relative fitting error for all data points was calculated. It was defined as the ratio of absolute difference between the loss density values calculated from the Equation (2) and the actual measured loss density values, referred to the actual measured values. It can be stated, that independently from the considered flux density and frequency range, the values of relative errors are smaller for the case, when the Equation (2) is determined on the basis of extended number of measurement data, including the additionally generated data points. The results are given in Table 2.

Table 2. The values of relative error  $\delta$  of fitting the Equation (2) to measurement data for a GO steel grade 111-35-N5 and a NO grade V350-50A

$B_m$ [T]	Steel grade 111-35-N5		Steel grade V350-50A	
	$\delta$ [%]	$\delta^x$ [%]	$\delta$ [%]	$\delta^x$ [%]
0.1-0.6	95	7	17	7
0.1-1.2	43	20	10	8
0.1-1.8	27	15	17	12
$f$ [Hz]	$\delta$ [%]	$\delta^x$ [%]	$\delta$ [%]	$\delta^x$ [%]
4-50	92	9	28	16
4-300	53	6	14	12
4-500	27	15	17	12

$\delta^x$  – relative error with additionally generated data points

Exemplary dependencies  $P(f)$  and  $P(B_m)$  for non-oriented sheet V350-50 A are depicted in Figures 3 and 4. In Figure 3 the dependency  $P(f)$  is shown for a wider range of independent quantities, than the one required by EN 10106 standard. This approach is justified by the increasing role of frequency converters as supply devices of electric motors, whose essential core parts are magnetic materials. When supplying an electric motor from a power-electronics frequency converters, an additional loss increase in the magnetic circuit caused by higher harmonics should be taken into account [10].

It should be remembered that novel soft magnetic materials, e.g. amorphous ribbons [11] or magnetodielectrics [12, 13], reveal much better properties than conventional FeSi steels for increased excitation frequencies.

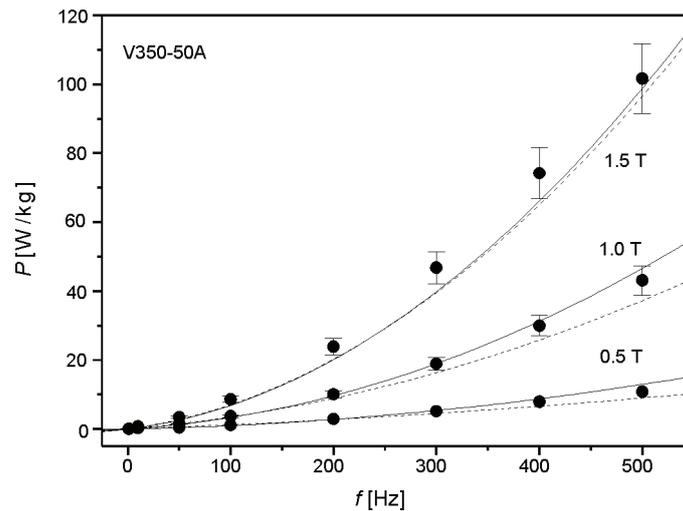


Fig. 3. Dependency  $P(f)$  for non-oriented sheet V350-50A. Dashed line – Equation (2) without additionally generated data points, solid line – Equation (2) with additional data points, dots – measurement points

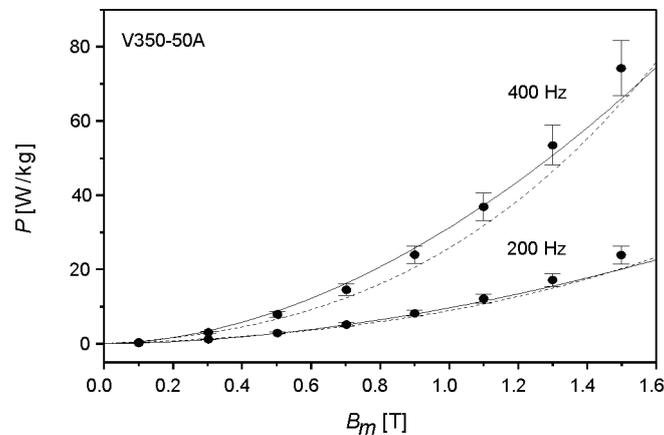


Fig. 4. Dependency  $P(B_m)$  for non-oriented sheet V350-50A. Dashed line – Equation (2) without additionally generated data points, solid line – Equation (2) with additional data points, dots – measurement points

It can be stated that the bootstrap method used for generation of additional data points makes it possible to improve the prediction accuracy of the abovementioned dependencies. The decrease of prediction error is important at the design stage of magnetic circuits, as their over-dimensioning may be avoided.

## 5. Conclusions

In the paper a method aimed at improving the fitting accuracy of the proposed loss density dependencies on maximum flux density and frequency is presented. For two typical grades of electrical steels (grain-oriented and non-oriented ones) a substantial accuracy improvement for the estimated parameters was achieved. It was shown that the values of errors between the measurement data points and those obtained from the proposed relationship are significantly lower for the case, when the fitting is supplemented with additional data points generated with the proposed bootstrap algorithm, in particular for lower values of flux density and frequency. Magnetic measurements carried out in this range are highly sensitive against measurement noise.

The presented relationship makes it possible to predict loss density in a wider range of frequency and flux density than Bertotti's formula. In the forthcoming research, an attempt to apply the presented model to describe the properties of soft magnetic composites shall be undertaken.

## References

- [1] Bertotti G., *General properties of power losses in soft ferromagnetic materials*. IEEE Transactions on Magnetics 24: 621-630 (1998).
- [2] Bertotti G., *Hysteresis in magnetism*. Academic Press, San Diego (1998).
- [3] Broddefalk A., Lindenmo M., *Dependence of the power losses of a non-oriented 3% Si-steel on frequency and gauge*. Journal of Magnetism and Magnetic Materials 304: e586-588 (2006).

- 
- [4] Sokalski K., Szczygłowski J., *Formula for Energy Loss in Soft Magnetic Materials and Scaling*. Acta Physica Polonica A 115: 920-924 (2009).
  - [5] Sokalski K., Szczygłowski J., Najgebauer M., Wilczyński W., *Losses scaling in soft magnetic materials*. COMPEL 26: 640-649 (2007).
  - [6] Szczygłowski J., Kopciuszewski P., Chwastek K. et al., *An improvement of accuracy of loss prediction in soft magnetic materials* (in Polish). Przegląd Elektrotechniczny 4: 45-47 (2010).
  - [7] Bajorek J., <http://www.rjmeasurement.com.pl> (2010).
  - [8] Efron B., *An introduction to the Bootstrap*. Chapman & Hall/CRC Monographs on Statistics & Applied Probability (2003).
  - [9] Chernick M.R., *Bootstrap Methods: A Guide for Practitioners and Researchers*. Wiley Series in Probability and Statistics (2007).
  - [10] Azarewicz S., Węgliński B., *Parametry wybranych blach prądnicowych przy podwyższonej częstotliwości magnesowania (Parameters of chosen generator sheets at elevated frequency of remagnetization – in Polish)*. Zeszyty Problemowe – Maszyny Elektryczne 80: 19-22 (2008), <http://www.komel.katowice.pl>
  - [11] Moses A., Leicht J., Fox D., *Influence of geometry and wave shape on magnetic amorphous material*. Journal of Magnetism and Magnetic Materials 290-291: 1520-1523 (2005).
  - [12] Saito T., Takemoto S., Iriyama T., *Resistivity and core size dependencies of eddy current loss for Fe-Si compressed cores*. IEEE Transactions on Magnetics 41: 3301-3303 (2005).
  - [13] Shokrollahi H., Janghorban K., *Soft magnetic composite materials (SMCs)*. Journal of Materials Processing Technology 189: 1-12 (2007).