www.czasopisma.pan.pl PAN www.journals.pan.pl

Arch. Min. Sci. **64** (2019), 3, 533-546

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.24425/ams.2019.129367

XIUFENG ZHANG*,**, GANQIANG TAO**#, ZHONGHUA ZHU**

A GRAVITY FLOW MODEL OF FRAGMENTED ROCKS IN LONGITUDINAL SUBLEVEL CAVING OF INCLINED MEDIUM-THICK ORE BODIES

MODEL PRZEPŁYWU FRAGMENTÓW SKALNYCH POD WPŁYWEM SIŁ CIĘŻKOŚCI PRZY EKSPLOATACJI PODPOZIOMOWEJ ZŁOŻA RUDY O ŚREDNIEJ MIĄŻSZOŚCI

The draw theory is the foundation for decreasing ore loss and dilution indices while extracting deposits from mines. Therefore, research on draw theory is of great significance to optimally guide the draw control and improve the economy efficiency of mines. The laboratory scaled physical draw experiments under inclined wall condition conducted showed that a new way was proposed to investigate the flow zone of granular materials. The flow zone was simply divided into two parts with respect to the demarcation point of the flow axis. Based on the stochastic medium draw theory, theoretical movement formulas were derived to define the gravity flow of fragmented rocks in these two parts. The ore body with 55° dip and 10 m width was taken as an example, the particle flow parameters were fitted, and the corresponding theoretical shape of the draw body was sketched based on the derived equation of draw-body shape. The comparison of experimental and theoretical shapes of the draw body confirmed that they coincided with each other; hence, the reliability of the derived equation of particle motion was validated.

Keywords: ore draw, sublevel caving, stochastic medium draw theory, inclined medium-thick ore body, draw body

Teorie dotyczące urabiania skał stanowią podstawę do działań mających na celu zmniejszenie wielkości strat rudy i wartości opisujących je współczynników w trakcie wybierania złóż. Dlatego też prowadzenie prac teoretycznych ma kluczowe znaczenie dla opracowania optymalnej strategii wybierania i poprawy ogólnej wydajności kopalni. Przeprowadzone eksperymentalne prace wydobywcze w wyrobiskach nachylonych prowadzone w warunkach laboratoryjnych wskazały celowość badania stref przepływu materiałów ziarnistych. Strefę przepływu podzielono na dwie części odpowiednio umiejscowione względem punktu demarkacyjnego na osi przepływu. W oparciu o teorie urabiania dla ośrodka stochastycznego wyprowadzono odpowiednie równania ruchu opisujące przepływ rozdrobnionego materiału skalnego w obydwu tych strefach pod wpływem sił ciężkości. Jako przykład rozpatrywano złoże rudy o nachyleniu 55° i szerokości

* COLLEGE OF RESOURCE AND CIVIL ENGINEERING, NORTHEASTERN UNIVERSITY, SHENYANG, LIAONING 110819, CHINA

** SCHOOL OF RESOURCE & ENVIRONMENT AND SAFETY ENGINEERING, UNIVERSITY OF SOUTH CHINA, HENGYANG, HUNAN 421001, CHINA

Correspondence: nhutgq@126.com

10 m, parametry przepływu cząstek skalnych zostały dobrane drogą dopasowania a prognozowany profil złoża wykreślono w oparciu o odpowiednie równania kształtu. Porównanie przewidywanego i rzeczywistego kształtu profilu złoża wykazało ich dużą zbieżność, tym samym potwierdzając wiarygodność opracowanego równania ruchu cząstek skał.

Słowa kluczowe: eksploatacja rud, eksploatacja podpoziomowa, teoria urabiania ośrodka stochastycznego, nachylone złoże o średniej miąższości, profil złoża

1. Introduction

There are some aspects, which include the changing trends in the isolated extraction zone (IEZ, where the original locus of extracted material, which is also named as draw body, associates with an individual drawpoint) (Melo, 2007; Pierce, 2010), the gravity flow mechanisms of blasted ore and waste rock, as well as the ore dilution and loss indices, closely associated with the stope boundary conditions. To achieve the highest productivity and the lowest operation cost in underground mining, the stope boundary has been a key research area in ore extraction (Janelid & Kvapil, 1966; Laubscher, 1994; Castro et al., 2014; Castro and Pineda, 2015; Sun et al., 2015). The boundary conditions in ore mining by the caving method can be divided into three categories according to the characteristics of the stope boundary (Ren, 1994). The first is the infinite boundary, which has no effect on the gravity flow of the caved materials. The second is the semi-infinite (i.e., end wall) boundary and the last is the inclined wall boundary, which affects the gravity flow of the particles. To date, many studies have been conducted on the ore mining process in these three boundary conditions by theoretical analyses, experimental research, and numerical simulations.

The conventional methods of theoretical analysis mainly are based on the ellipsoid draw theory (Малахов, 1958; Kvapil, 1965a; Liu, 1979; Li, 1983) and stochastic medium draw theory (Litwiniszyn, 1956; Mullins, 1972, 1974; Ren, 1992, 1994; Chen, 1997; Qiao, 2006). These theories have been verified by a number of experimental studies conducted by various scholars (Zhou, 2006; Wang, 2015). They have also been widely applied in situ due to the availability of high degree of simulation and the effectiveness in solving practical problems. However, the geometrical draw shape was delimitated with the ellipsoidal draw theory, but the caving mechanism could not be provided. Moreover, the laboratory experiments and in situ observations indicated that the shape of the draw body was not exactly ellipsoidal (Chen, 1997; Rustan, 2000; Kuchta, 2012; Jin, 2017). Litwiniszyn (1956) proposed a stochastic model to research the flow of dry sand. Ren (1992, 1994) proposed a new material flow model and established a probability density function to represent the granular flow.

Ren (1994) implemented the piecewise function to derive equations to represent the particle motion in inclined wall boundary condition of an ore body with inclination above 60°. Huang and Zhao (1986) considered an ore body with a dip of 65° for the research, studied the basic flow of fragmented rocks, proposed a combinatorial theory with two mathematical shapes, and established the corresponding mathematical model.

Experimental investigations, including in situ and laboratory model extraction, are critical in research on ore extraction, and abundant experiments have been conducted (Janelid, 1972; Kvapil, 1965b; Stazhevskii, 1996; Brunton et al., 2010; Ren et al., 2018). For instance, Castro et al. (2007) studied the flow mechanisms of cohesionless particles and the effect of some factors on the isolated draw zone with a large 3D physical model. Aiming at the inclined medium-thick ore body in Xiadian gold mine that was extracted by sublevel caving, Zhou (2006) improved the stope structural parameters and the mode of blasting and extracting. Wang (2015) performed model draw experiments to study the shape of IEZ for a tilt-steeply inclined, medium-thick ore body with 58° inclined wall angle and 15 m ore width. Additionally, the results from many laboratory draw experiments conducted with ore, sand, or gravel as the medium promoted rapid research development on the flow laws of granular material (Waston, 1993; Power, 2004; Trueman et al., 2008; Castro, 2006; Castro & Pineda, 2015).

Numerical simulations have also been widely applied to study the ore extraction (Jolley, 1968; Hancock, 2013; Irazábal, 2017; Lapčević & Torbica, 2017; Xu et al., 2017). For example, it is convenient and flexible to conduct draw simulations and analyse the flow mechanisms of blasted ore and waste rock and the dynamic responses of particles by means of some simulation technique for ore draw, such as the Particle Flow Code, and the Discrete Element Method (Cundall & Strack, 1979; Cleary & Sawley, 2002; Pierce et al., 2002; Minchinton & Dare-Bryan, 2005; Jin et al., 2016). Pierce (2010) directed towards the development of a model for gravity flow of fragmented rock and embedded the understanding of gravity flow under some conditions presented in a caving mine into REBOP (i.e. Rapid Emulator Based on PFC). Sun et al. (2015) conducted isolated physical draw experiments and constructed numerical models using PFC to investigate the shape of IEZ as well as its major influencing factors, and formulated the flow laws of particles under inclined wall boundary condition.

In terms of the third type boundary condition, namely, the inclined wall boundary mentioned above, the ore draw under this condition is more common in the field, and the phenomenon that ore loss and dilution exist in the process of drawing is more serious in actual production. High dilution and loss rates can result in the wastage of mine resources and pollution of the surrounding environment. Since the theoretical analysis about the gravity of particles with regard to the inclined medium thick ore body is not so much far, the basic equations about granular particles flow laws were deduced on the basis of Ren's stochastic medium draw theory under the premise of laboratory scaled physical experiments conducted with a geometrical scale 1:25 under inclined wall boundary condition in this study. Note that there were two factors, which were the dip (45° ~ 55°) and the width of ore body (4 m ~ 10 m) respectively, mainly taken into consideration.

2. Descriptions of laboratory physical model draw experiment

2.1. Experimental Model

In the caving method of mining, inclined wall boundary is defined as the condition where the gravity flow of the blasted ore and waste rock is influenced by the inclined wall; this occurs when the dip of the ore body is less than 90° and greater than the material's natural repose angle (Ren, 1994). Fig. 1 illustrates the scaled physical model designed to investigate the gravity flow of blasted ore and waste rock in sublevel caving, by the isolated draw tests under inclined wall boundary condition (Zhang et al., 2018). The geometric scale of the model was considered as 1:25, and the size and shape of the model, ore size, and other geometric parameters were determined based on the similarity principle.

Three different angles of inclined wall (45°, 50°, and 55° dip) and three different widths of ore body (scaled to 16, 24, 32, and 40 cm) were considered in the tests to investigate the influ-

Fig. 1. Scaled physical model for ore draw under inclined wall boundary condition

ences on the shape of IEZ. Additionally, each scheme was repeated thrice to reduce the influence of random movement of particles on the gravity flow.

The material used in this study was magnetite ore derived from a local iron mine in Hunan province, China, and the largest block size was 500 mm. The material was broken in the laboratory into particles of size less than 20 mm, based on the similitude principle. Additionally, the bulk density of magnetite ore was 1841 kg/m^3 and material's natural repose angle was 33.4°. To obtain the precise coordinates and corresponding discharged quantity for delineating the shape of IEZ, the method of placing labelled particles was adopted (Tao et al., 2009). The labelled markers were distributed every 50 mm in the height direction of the model, and a row of marked particles was placed in the centre of each horizontal level at a spacing of 15 mm. These particles were directly selected from the extracted material to maintain the flow behaviours of labelled markers in accordance with those of the experimental material, and each marked particle was coloured and labelled to indicate a coordinate point in the model.

2.2. Experimental Results

A demarcation point in the flow axis ($z = z_L$) was identified based on the configuration of the vertical velocity distribution curve of the particles, the relationship between the relative positions of points with the fastest flow, and the axis of the isolated drawpoint. The analysis of the experimental results revealed that the upper flow axis was parallel to the upper wall and the lower flow axis coincided with the central axis of the drawpoint (Zhang et al., 2018). Therefore, for simplicity, the flow zone of granular materials was proposed to be divided into two parts with respect to the demarcation point.

The two parts of the flow zone (Zone A and B) are illustrated in Fig. 2, where θ represents the inclined wall angle. Zone A is the area where the gravity flow is unaffected by the inclined upper wall (i.e. the free area), and the laws of particle motion in it is similar to that of the infinite boundary condition; Zone B is the area where the gravity flow is influenced by the inclined upper wall boundary condition.

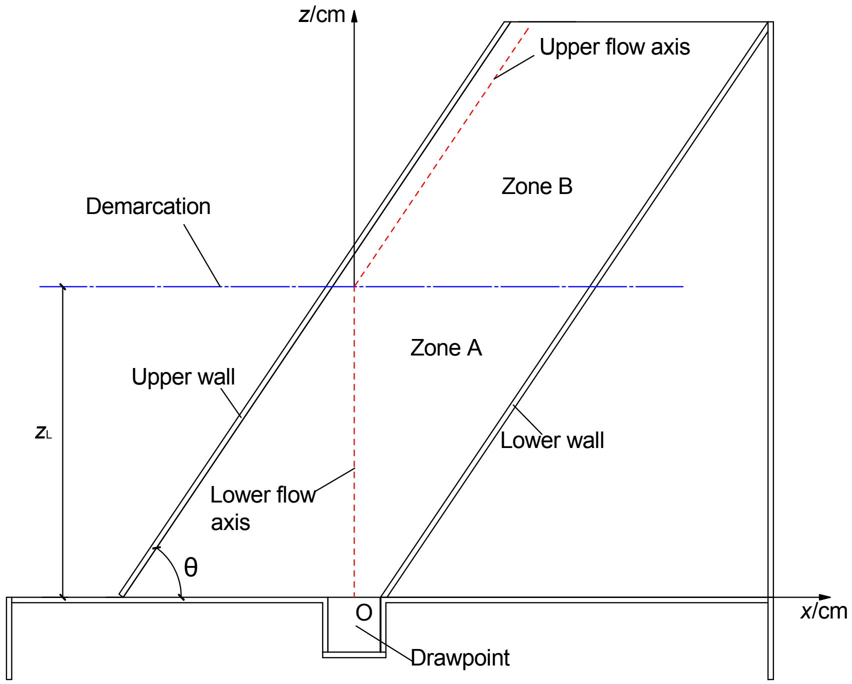

Fig. 2. The division of flow zone for ore draw under inclined wall condition

3. Stochastic medium theory for ore extraction under inclined wall condition

3.1. Probability density function of granular particle movement

The blasted ore and waste rock are particles with complex porous structure. As they encounter appropriate spatial conditions, the particles will flow to the drawpoint under gravity. Suppose the effect of instantaneous loose property is not taken into account during the process of flowing, these particles can be simplified as a continuous flow media. Conventional stochastic medium theory could explain the phenomenon of ore draw; however, the macroscopic restriction by the effect of movement field on the particle flow pattern was neglected. Therefore, Ren proposed the modified stochastic medium draw theory (Ren, 1994), where the internal flow law of particle is defined from a statistical perspective that the coordinates of particles will change from the point of smaller probability to the larger one from a statistical perspective. Physical simulation experiments were conducted to observe the basic gravity flow of particles and to study the shape of the IEZ. However, the experiment had only one draw step, and there were no labelled markers in the depth direction (y-axis) of the scaled model. Therefore, a mathematical model of two dimensions was established in this study by combining the theoretical analysis and the laboratory experiments of ore extraction.

The probability density function of the granular flow of particles in the area unaffected by the inclined upper wall (i.e. $z \le z_L$) can be represented as,

$$P(x,z) = \frac{1}{\sqrt{\pi\beta z^{\alpha}}} e^{-\frac{x^2}{\beta z^{\alpha}}} \tag{1}$$

where α, β are the flow parameters of particles in Zone A, and x, z are the continuous variables denoting the coordinates of particles.

To represent the effects of the inclined wall boundary condition on the particle flow, a constant f, a variable-influencing coefficient, is introduced to the probability density function of the granular flow in the area influenced by the inclined upper wall (i.e. $z > z_L$), which can be represented as,

$$P(x,z) = \frac{1}{f\sqrt{\pi\beta_1 z^{\alpha_1}}} e^{-\frac{(x-u)^2}{\beta_1 z^{\alpha_1}}} \tag{2}$$

where α_1, β_1 are the flow parameters of particles in Zone B, u is the horizontal distance from the upper flow axis to z-axis in this area, and θ is the angle of the inclined wall (i.e., the dip of the ore body). Thus, $u = (z - z_L)\cot\theta$, and f is defined as the vertical distance from the upper flow axis to the proximate inclined wall (i.e. the upper inclined wall).

3.2. The equation of particle movement velocity

Presuming that q is the mass of particles extracted from the isolated drawpoint (mass drawn) in a unit time, the velocity of vertical descent of particles, v_z at any point $P(x, z)$ in the flow zone can be represented as,

$$v_z = -qP(x,z) \tag{3}$$

where the negative sign signifies that the direction of velocity is opposite to the direction of positive coordinate.

To ensure a continuous flow, the suppose that the inflow of particles should be equal to the outflow for a unit time was made. Therefore, at any point in the flow zone, the particle movement velocity should satisfy the passive field continuous flow equation, which is

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \tag{4}$$

where v_x and v_z denote the particle velocities in the horizontal and vertical directions, respectively.

Eqs. (3) and (4) can be combined and written as, when $z \le z_L$,

$$v_x = -\frac{q\alpha x}{2\sqrt{\pi\beta z^{\alpha+2}}} e^{-\frac{x^2}{\beta z^{\alpha}}} \tag{5}$$

and when $z > z_L$,

$$v_x = -\frac{q\left[2z\cot\theta + \alpha_1\left(x-u\right)\right]}{2f\sqrt{\pi\beta_1 z^{\alpha_1+2}}} e^{-\frac{(x-u)^2}{\beta_1 z^{\alpha_1}}} \tag{6}$$

3.3. The equation of particle movement trace

The curve form of particle movement trace is determined by the distribution of velocity field. The laws of physics imply that the tangent line at any fixed point (x, z) on the particle movement trace is collinear with the direction of particle movement velocity. Therefore, when $z \leq z_L$,

$$\frac{dx}{dz} = \frac{v_x}{v_z} = \frac{\alpha x}{2z}$$

when $z > z_L$,

$$\frac{dx}{dz} = \frac{v_x}{v_z} = \cot\theta + \frac{\alpha_1\left(x-u\right)}{2z}$$

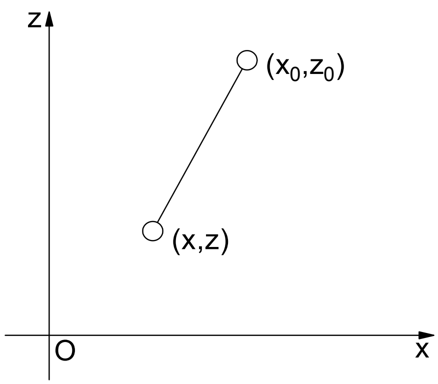

Fig. 3. Schematic diagram of particle movement

The particle motion thus obtained is illustrated in Fig. 3. Integrating the above equations representing the deduced trace curves of the two zones, the particle movement trace equations, given below, can be obtained.

When $z \leq z_L$,

$$\frac{x^2}{z^\alpha} = \frac{{x_0}^2}{{z_0}^\alpha} \tag{7a}$$

When $z > z_L$,

$$x = x_0 + \int_{z_0}^{z} \cot\theta + \frac{\alpha_1 (x-u)}{2z} \mathrm{d}z \tag{7b}$$

where x_0, z_0 and x, z are the coordinate values that represent the anterior and present positions of particles, respectively.

3.4. The drawn-funnel equation

The drawn-funnel can be described as the geometric shape obtained when all the particles at a level move simultaneously to a new position. When the particles are extracted from an isolated drawpoint at the bottom, they move to the drawpoint at a velocity of $\vec{v} = \vec{v}_x + \vec{v}_z$ along the locus curve. Along the direction of the axis, the value of particle movement velocity is equal to the rate of change of coordinates with respect to time, i.e., $\mathrm{d}_z/\mathrm{d}_t = v_z$. Substituting Eq. (3) into this equation, integrating along the particle trace curve, and considering Eqs. (7a) and (7b), the relationship between the mass drawn and particle position can be represented as,

$$Q_f = qt = -\int_{z_0}^{z} \frac{1}{P(x,z)} \mathrm{d}z \tag{8}$$

where Q_f is the total mass drawn, and t is the time of draw.

Given the mass drawn at a level z_0, the new position (x, z) of each particle A_0 (x_0, z_0) can be calculated using Eqs. (8) and (7a, b), and the drawn-funnel can be plotted.

3.5. The equations of discharged quantity and IEZ's shape

With an increase in the mass drawn, the particles continuously approach the drawpoint. When the particles at point A_0 (x_0, z_0) reaches the drawpoint (i.e. $z = 0$), the corresponding mass drawn, Q_0, is regarded as the discharged quantity of A_0. Substituting $z = 0$ into Eq. (8), the equation of discharged quantity can be obtained as,

$$Q_0 = -\int_{z_0}^{0} \frac{1}{P(x,z)} \mathrm{d}z \tag{9}$$

Thus, the expression of the discharged quantity in the area unaffected by the inclined wall can be deduced from Eq. (9) as,

$$Q_0 = \frac{2}{\alpha+2} \sqrt{\pi\beta}\, z_0^{\frac{\alpha}{2}+1} e^{\frac{x_0^2}{\beta z_0^{\alpha}}}$$

Similarly, the equation of discharged quantity in the area influenced by the inclined wall can be deduced as,

$$Q_0 = Q_L + \sqrt{\pi\beta_1} \int_{z_L}^{z_0} f \sqrt{z^{\alpha_1}}\, e^{\frac{(x-u)^2}{\beta_1 z^{\alpha_1}}} \mathrm{d}z \tag{10}$$

Moreover, in Eq. (10), Q_L is

$$Q_L = \frac{2}{\alpha+2}\sqrt{\pi\beta}\, z_L^{\frac{\alpha}{2}+1} e^{\frac{x_L^2}{\beta z_L^{\alpha}}}$$

where x_L is the coordinate of the particle when the point $A_0(x_0, z_0)$ moves to the level $z = z_L$.

The IEZ is the original locus of the extracted material associated with an isolated drawpoint. The surface of IEZ is the isosurface of the discharged quantity field, so the shape of IEZ for the two parts can be expressed as,

When $z \le z_L$,

$$x^2 = \frac{\alpha+2}{2}\beta z^{\alpha} \ln\frac{H}{z} \tag{11}$$

When $z > z_L$,

$$\int_{z_L}^{z} f\sqrt{z^{\alpha_1}}\, e^{\frac{(x-u)^2}{\beta_1 z^{\alpha_1}}}\,\mathrm{d}z = \left(Q_F - Q_L\right)\frac{1}{\sqrt{\pi\beta_1}} \tag{12}$$

$$\left(x = x_L + \int_{z_L}^{z} \cot\theta + \frac{\alpha_1 (x-u)}{2z}\mathrm{d}z\right)$$

where H is the height of draw.

4. Validation and analysis

The analysis demonstrates that the particles on the surface of the draw body are drawn simultaneously from the drawpoint; therefore, the shape of IEZ is the isopleth of the discharged quantity. Hence, the discharged quantity of each extracted labelled marker should be noted during the tests by recording the number and weighing the mass drawn each time. To delimitate the shape of IEZ under inclined wall condition, this discharged quantity method is usually applied in scaled physical model draw experiments. In this method, firstly, a function is established using the least square method to represent the relationship between the coordinate of each labelled marker and the corresponding discharged quantity for each height of draw, and then the discharged quantity is obtained. Secondly, given the minimum discharged quantity at a particular height of draw, the corresponding coordinates of particles with the same discharged quantity for other heights of draw is calculated by interpolation. Lastly, the experimental shape of IEZ can be delimitated to a smooth curve using these coordinates.

The accuracy of the above theoretical equations was verified based on the experimental data in the tests and the non-linear Levenberg-Marquardt regression method by calculating the fitting values of the flow parameters and the corresponding correlation coefficients of the two parts, as shown in Table 1. The values of the correlation coefficients are greater than 0.9, which indicate that the experimental data fit the theoretical equations well, given the laboratory experimental condition; the correlation between the values of flow parameters is reasonable. For the Zone A,

TABLE 1

Flow Parameters and corresponding correlation coefficients of two parts in this study

Dip of ore body, °	Width of ore body, cm	Zone A ($z \le z_L$)			Zone B ($z > z_L$)			
		α	β	r	α_1	β_1	f	r
55	40	1.427	0.663	0.955	0.420	1.027	3.40	0.989
55	32	1.039	2.494	0.947	0.481	0.982	3.59	0.969
55	24	1.151	1.523	0.935	0.326	0.974	3.51	0.973
55	16	0.685	5.761	0.990	0.234	0.955	3.02	0.993
50	40	1.418	0.683	0.952	0.479	0.969	2.64	0.955
45	40	1.334	0.960	0.910	0.468	0.864	2.92	0.964

parameters can be obtained directly using the experimental data and Eq. (11). However, the integrand expression in Eq. (12) for Zone B is relatively complicated, so the parameters of this zone can be calculated by combining Eq. (12) with the discharged quantity method.

Similarly, the theoretical shape of IEZ in Zone A can be plotted easily and directly by substituting the flow parameters α, β from Table 1 into Eq. (11). However, for Zone B, based on the method of particle movement trace, the coordinates of points with the value of Q_f equal to the mass drawn at a certain level can be found along each curve of the particle movement trace. Hence, the shape of IEZ in Zone B can be drawn by connecting these points by a smooth curve.

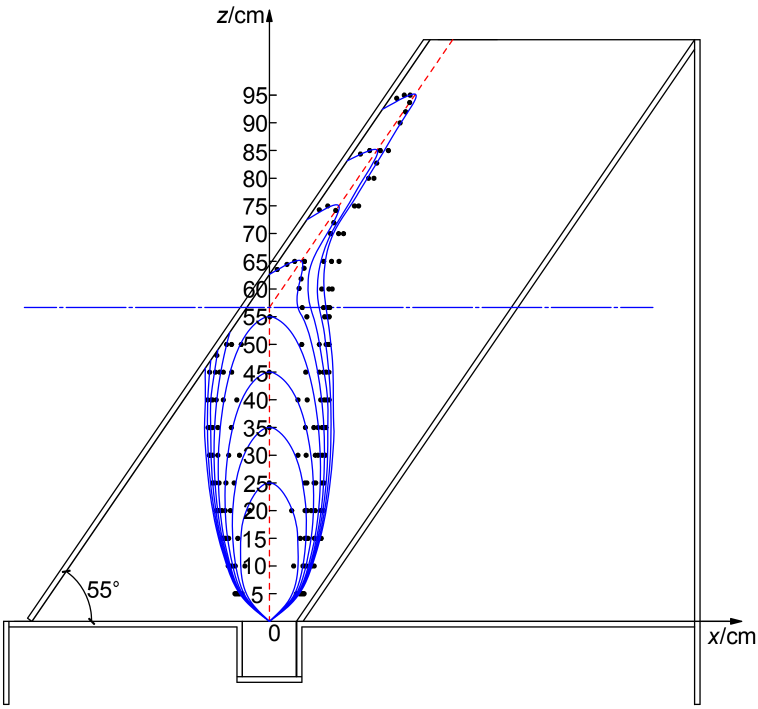

Fig. 4. Comparison of IEZ's shape (• represent the experimental data, — represent the theoretical value)

Finally, the complete theoretical shape of IEZ under the condition of inclined wall can be drawn by combining these two.

Making a comparison between the experimental values and theoretical shapes of IEZ, both are ultimately in good conformity with each other. Obviously, the accuracy of theoretical equations can be verified. Therefore, an ore body with the angle of 55° and the width of 40 cm in this study was taken as an example. The results are depicted in Fig. 4, where the shape of the IEZ from the experiment is represented by the critical points (experimental data) at different levels and the theoretical shape is displayed by the smooth curve. Therefore, compared to the conventional division to three flow parts (Ren, 1994), the improved division to two parts, proposed in this study, is feasible and accurate, and it can be convenient and appropriate for researching the gravity flow of blasted ore and waste rock under inclined wall boundary condition.

5. Discussion

In actual mines, the inclined wall boundary condition is more common in general. The draw theory for complex conditions has been a difficult topic in theoretical research field because the shape of the draw body is no longer symmetrical and same. However, a model to analyse the shape of the draw body has an important design implication in the mines. Moreover, in China, the inclined medium-thick ore bodies account for approximately one-fifth of the total non-ferrous metal ores in underground mines. In sublevel caving, the blasted ore, surrounded by the overlying waste rock, is extracted from the sublevel drifts, where the waste rock is caved spontaneously under gravity. This indicates that more ore can be extracted if the extraction of the waste materials are controlled and regulated. The draw theory is the precondition and foundation for comprehending the flow laws of fragmented material. Therefore, the main aim of this paper was to research the theoretical model of draw body and broaden the scope of the present draw theory, based on the stochastic medium draw theory.

In the experiments, only two main actors were considered to imitate the inclined medium-thick ore body; therefore, the influence of the drawpoint on the mass drawn was not taken into account. However, the effect of size of the drawpoint on the mass drawn will not change the mathematical model, but influence the corresponding parameters.

The laboratory results demonstrated that the inclined wall changed the flow direction of particles. A fundamental parameter, the relative relation of time required for the particles to move downward to the drawpoint, had changed, resulting in a huge variation in the shape of IEZ with increase in the height of draw. The draw body became more elongated when the height of draw increased. It is also possible to determine the volume of ore that can be drawn before the waste rock reaches the drawpoint. Additionally, the laboratory results indicated that the proposed method is a simplified mean to research the gravity flow for inclined medium-thick ore body, when compared with the existing models. Moreover, the simplified division was used as the basis for the theoretical analysis, and the formulations were deduced according to the probability density function of granular particle movement.

The draw theory must be implemented in practice to reveal the movement field and to reflect the fidelity in the shape of IEZ. Although the scaled physical draw model cannot completely mimic the actual stope, the draw simulation satisfies the practical requirements because the model is similar to the shape of the draw body in geometry and the initial and boundary conditions are met as far as possible. Moreover, the mathematical model proposed in this study has demonstrated good agreement with the experimental results, in general and thus provides a better and simpler description to analyse the adaptability of the stope structural parameters in sublevel caving method and draw techniques with gravity flow of particles.

In this paper, these results are obtained by a simple isolated draw test that is easy to understand. However, further research should be conducted to understand the practical implications of the proposed simplified division of the particle flow zone and gravity flow of blasted ore and waste rock for the inclined medium-thick ore body. In addition, the equation of the shape of IEZ, derived based on stochastic medium draw theory, should be researched further to understand its applicability in an actual mine.

6. Conclusions

Based on the results from the physical draw experiments on the scaled models, a new, simplified division of the particle flow zone was proposed. Based on this division, the equations of particle movement velocity, particle movement trace, drawn-funnel, and the shape of draw body were deduced from the probability density function of granular particle movement. In this paper, the theoretical research on the shape of IEZ is based on the stochastic medium draw theory, which is a conventional theory used to study the shapes of IEZ and investigate the flow laws of blasted ore and waste rock in caving method of mining. Moreover, it is of great importance to help forecast the ore loss and dilution and to optimize the stope structural parameters. Besides, in order to verify the accuracy and rationality of the derived equations, the theoretical shape obtained from these equations were compared with the experimental shapes of IEZ, drawn by the method of discharged quantity. The characteristics of flow axis and experimental shapes of IEZ confirmed the reliability and simplicity of the proposed division method of the flow zone. Therefore, this reference, to a certain extent, can be used in the theoretical research on ore draw under complex boundary conditions as well.

Acknowledgments

This work was supported by the National Natural Science Foundation of P. R. China with Grant No.51574151.

References

Brunton I., Fraser S., Hodgkinson J., Stewart P., 2010. *Parameters influencing full scale sublevel caving material recovery at the Ridgeway gold mine*. Int. J. Rock Mech. Min. Sci. **47**, 647-656.

Castro R., 2006. *Study of the Mechanisms of Granular Flow for Block Caving*. University of Queensland.

Castro R., Trueman R., Halim A., 2007. *A study of isolated draw zones in block caving mines by means of a large 3D physical model*. Int. J. Rock Mech. Min. Sci. **44**, 6, 860-870.

Castro R., Pineda M., 2015. *The role of gravity flow in the design and planning of large sublevel stopes*. J. S. Afr. Inst. Mini. Metall. 1**15**, 2, 113-118.

Castro R.L., Fuenzalida M.A., Lund F., 2014. *Experimental study of gravity flow under confined conditions*. Int. J. Rock Mech. Min. Sci. **67**, 164-169.

Chen G., 1997. *Stochastic modelling of rock fragment flow under gravity*. Int. J. Rock Mech. Min. Sci. **34,** 2, 323-331.

Cleary P.W., Sawley M.L., 2002. *DEM modelling of industrial granular flows: 3D case studies and the effect of particle shape on hopper discharge*. Appl. Math Modell. **26**, 2, 89-111.

Cundall P.A., Strack O.D., 1979. *A discrete numerical model for granular assemblies*. Géotechnique **29**, 1, 47-65.

Hancock W., 2013. *Gravity flow of rock in caving mines: numerical modelling of isolated, interactive and non-ideal draw*. Dissertation, The University of Queensland, Brisbane, Australia.

Huang X., Zhao Q.H., 1986. *A numerical analog study on drawing in caving with inclined walls*. Nonferrous Met **38**, 2, 1-6.

Irazábal J., Salazar F., Oñate E., 2017. *Numerical modelling of granular materials with spherical discrete particles and the bounded rolling friction model. Application to railway ballast*. Comput. Geotech. **85**, 220-229.

Janelid I., Kvapil R., 1966. *Sublevel caving*. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. **3**, 2, 129-132.

Janelid I., 1972. *Study of the gravity flow process in sublevel caving*. In Proceedings of the International Sublevel Caving Symposium, Stockholm, Sweden, 25-27 September; Atlas Copco: Stockholm, Sweden.

Jin A.B., Sun H., Ma G.W., Gao Y.T., Wu S.C., Meng X.Q., 2016. *A Study on the draw laws of caved ore and rock using the discrete element method*. Comput. Geotech. **80**, 59-70.

Jin A.B., Sun H., Wu S.C., Gao Y.T., 2017. *Confirmation of the upside-down drop shape theory in gravity flow and development of a new empirical equation to calculate the shape*. Int. J. Rock Mech. Min. Sci. **92**, 91-98.

Jolley D., 1968. *Computer simulation of the movement of ore and waste in an underground mining pillar*. Canada Mining and Metal Bulletin **675**, 854-859.

Kuchta M.E., 2012. *A revised form of the Bergmark-Ross equation for describing the gravity flow of broken rock*. Min. Resour. Eng. **11**, 349-360.

Kvapil R., 1965a. *Gravity flow of granular materials in hoppers and bins Part I*. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. **2**, 1, 25-34.

Kvapil R., 1965b. *Gravity flow of granular materials in hoppers and bins Part II. Coarse material*. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. **2**, 3, 277-304.

Lapčević V., Torbica S., 2017. *Numerical investigation of caved rock mass friction and fragmentation change influence on gravity flow formation in sublevel caving*. Minerals **7**, 4, 56.

Laubscher D.H., 1994. *Cave mining – The state of the art*. J. S. Afr. Inst. Min. Metall. **94**, 10, 279-293.

Li R.F., 1983. *The unified mathematical equations of the basic laws of ore drawing*. Nonferrous Met.(Min. Sect.) **1**, 1-7.

Litwiniszyn J., 1956. *Application of the equation of stochastic processes to mechanics of loose bodies*. Arch. Mech. **8**, 4, 393-411.

Liu X.G., 1979. *The basic movement law of caved ore and rock in caving mining (part I)-research on the ellipsoid theory*. Nonferrous Met. (Min. Sect.) **4**, 38-44).

Малахов Г.М., 1958. *Drawing of caved ore* (Yang QR, Liu XG Trans). Metallurgical Industry Press, Beijing, China.

Melo F., Vivanco F., Fuentes C., Apablaza V., 2007. *On drawbody shapes: from Bergmark-Ross to kinematic models*. Int. J. Rock Mech. Min. Sci. **44**, 1, 77-86.

Minchinton A., Dare-Bryan P., 2005. *The application of computer modelling for blasting and flow in sublevel caving operations*. In: Proceedings of the 9th AusIMM Underground Operators' Conference, Perth, Australia, 7-9 March, 65-73.

Mullins W.W., 1972. *Stochastic theory of particle flow under gravity*. J. Appl. Phys. **43**, 2, 665-678.

Mullins W.W., 1974. *Nonsteady state particle flow under gravity-an extension of the stochastic theory*. J. Appl. Physics. **41**, 4, 867-872.

Pierce M., Cundall P., Van Hout G., Lorig L., 2002. *PFC3D modeling of caved rock under draw*. In: Proceedings of the 1st International PFC Symposium, Gelsenkirchen, Germany, 6-8 November, 211-217.

Pierce M., 2010. *A model for gravity flow of fragmented rock in block caving mines*. Block Caving **54**, 2, 930-937.

Power G., 2004. *Modelling granular flow in caving mines: large scale physical models and full scale experiments*. Dissertation, University of Queensland.

Qiao D.P., 2006. *Research and application of draw theory*. Yunnan Science Press, Kunming.

Ren F.Y., 1992. *Draw theory and its application based on stochastic theory*. Dissertation, Northeastern University.

Ren F.Y., 1994. *The research and application of the stochastic medium theory for ore drawing*. Metallurgical Industry Press, Beijing.

Ren F.Y., Liu Y, Cao J.L., He R.X., Fu Y., Zhou Y.J., Liu H., 2018. *Prediction of the caved rock zones' scope induced by caving mining method* [J]. PLoS ONE, 13 (8): e0202221.

Rustan A., 2000. *Gravity flow of broken rock-what is known and unknown*. In: Proceedings of MassMin. Brisbane, 29 October-2 November, 557-567.

Stazhevskii S.B., 1996. *Features of flow of broken rock in extraction of ores with sublevel caving*. J. Min. Sci. **32**, 5, 403-416.

Sun H., Jin A.B., Gao Y.T., Zhou Y., Yang Z.W., 2015. *Flow characteristic of caved ore and rock under complex boundary condition*. J. Cent. South. Univ. (Sci. Technol.) **46**, 10, 3782-3788.

Tao G.Q., Yang S.J., Ren F.Y., 2009. *Experimental research on granular flow characters of caved ore and rock*. Rock Soil Mech. **30**, 10, 2950-2954.

Trueman R., Castro R., Halim A., 2008. *Study of multiple draw-zone interaction in block caving mines by means of a large 3D physical model*. Int. J. Rock Mech. Min. Sci. **45**, 7, 1044-1051.

Wang C., 2015. *Study on stope structure parameters of inclined and steeply inclined medium thick ore bodies in Jianshan mining area*. Dissertation, Kunming University of Science and Technology (in Chinese).

Watson G.R., 1993. *Flow patterns in flat bottomed silos*. Dissertation, University of Edinburgh.

Xu S., Suorineni F.T., An L., Li Y.H., 2017. *A study of gravity flow principles of sublevel caving method in dipping narrow veins*. Granular Matter **19**, 4, 82.

Zhang X.F., Tao G.Q., Zhu Z.H., 2018. *Laboratory study of the influence of dip and ore width on gravity flow during longitudinal sublevel caving*. Int. J. Rock Mech. Min. Sci. **103**, 179-185.

Zhou Z.H., 2006. *Research on low-dilution and loss sublevel caving of inclined medium-thick orebody in Xiadian gold mine*. Dissertation, Northeastern University.