www.czasopisma.pan.pl PAN www.journals.pan.pl
Metrol. Meas. Syst., Vol. 25 (2018) No. 4, pp. 659–673.

METROLOGY AND MEASUREMENT SYSTEMS
Index 330930, ISSN 0860-8229
www.metrology.pg.gda.pl

STRUCTURAL MODELLING OF THROTTLE DIAGRAMS FOR MEASURING FLUID PARAMETERS

Yevhen Pistun, Halyna Matiko, Hanna Krykh, Fedir Matiko

Lviv Polytechnic National University, Bandera 12, Lviv, 79013, Ukraine (*epistun@polynet.lviv.ua*, ✉ *halynamatiko@gmail.com*, +38 050 921 3651, *hannakrih@gmail.com*, *fmatiko@gmail.com*)

Abstract

In the paper there are presented tools for structural modelling of throttle diagrams that are developed as a basis to building transducers used for measuring fluid parameters. The definitions of throttle diagrams are improved and their classification is developed. Dependences are obtained to calculate the number of measuring channels in a throttle diagram and the number of possible variants of measuring transducers using the combinatory apparatus. A procedure for mathematical description of throttle diagrams in the form of graphs is proposed which makes it possible to obtain all diagrams with different measuring channels on the basis of certain throttle diagram. The model is developed in the form of a graph. A schematic diagram and a mathematical model of a transducer measuring physical and mechanical parameters of Bingham plastic fluid are developed based on a throttle diagram.

Keywords: throttle diagram, measuring transducer, model, fluid.

1. Introduction

Measuring transducers and systems with throttles as sensors are widely used for continuous measurement of physical and mechanical parameters of fluids. Throttles are the elements that create resistance to the flow of liquid or gas [1]. They are divided into laminar and turbulent ones depending on the regime of fluid steady motion. They are also divided into constant and variable throttles depending on the change of their hydraulic resistance during measurement, which is not caused by changes of liquid parameters.

The measurement method using throttle elements is called hydrodynamic or gas-dynamic depending on the dynamic processes that arise during the movement of liquid or gas through the throttle. The application of throttle elements of different types and dimensions which are arranged in parallel and in series, in bridge and differential measuring diagrams, enables to synthesize hydro-gas-dynamic transducers and systems for measuring physical and mechanical parameters of fluids with different transform functions and metrological characteristics [2].

Thus, in [3] a viscometer with a differential measuring diagram is considered. It is built on the basis of a series-parallel connection of tubes or capillaries. They form a hydraulic bridge equivalent to the Wheatstone bridge in electronics. A hydrodynamic transducer created on the basis of bridge diagram with laminar and turbulent throttle elements is used in an intelligent

Article history: received on May 17, 2018; accepted on Aug. 14, 2018; available online on Dec. 31, 2018, DOI: 10.24425/mms.2018.124884.

information-measuring system of automobile fuel quality [4]. The principle of measurement is in continuous pumping of the explored oil products through the transducer and automatic astatic balancing of the bridge by changing the volumetric flowrate of the product. Such parameters as kinematic viscosity, density, dynamic viscosity, octane number, cetane number and oil product index are determined with high accuracy by the values of the volumetric flowrate and the total pressure drop across the bridge at the moment of its balance. Similar transducers based on a bridge diagram with different types of throttle elements are also used for measuring the viscosity and density of emulsions [5]. In addition to traditional methods of balancing the viscometer bridge by changing the flowrate or length of one or more capillaries, another method is described in [6]. In this transducer the resistance of fluid flow is regulated by a mechanical device. The viscometer built on a capillary bridge in [7] is intended to measure a specific viscosity. The viscometer in [8] is built on the basis of a diagram with three different capillaries. It measures the relative viscosity by which the molecular weight of a polymer is determined. Throttle elements in the form of tubes are also used to measure the parameters of non-Newtonian fluids. Thus, in [9] methods and systems are considered for real-time determination of rheological parameters of liquids in wells in drilling operating conditions.

The papers [10, 11] show that gas-dynamic transducers can be used for measuring micro flowrate of gases (below $3.0 \cdot 10^{-6}$ m^3/s) as well as small flowrate of gases (from $3.0 \cdot 10^{-6}$ to $3.0 \cdot 10^{-3}$ m^3/s). The throttle elements of such transducers form the branches of a bridge measuring diagram which provides high sensitivity and accuracy of flowrate measurement. In [12], an analyser of nitrogen-hydrogen gas mixture is proposed to be built on the basis of a gas-dynamic bridge diagram. It is shown that the characteristics and functionality of the analyser depend on various combinations and the numbers of laminar and turbulent throttles in the bridge diagram.

It should be noted that the theory of synthesizing the measuring transducers based on one throttle element is well developed for many practical applications. Thus, in a variable differential pressure flowmeter the fluid flowrate is determined by the pressure drop across the primary device (throttle element) mounted in a pipeline [13]. The international standards [14] show the achieved level of flowrate measurement by using the differential pressure transducers with various primary devices. The computer-aided design of differential pressure flowmeters is developed in [15] on the basis of these standards.

The connection of several identical or different types of elements in throttle diagrams in different ways and the possibility of choosing different measuring channels create a wide range of diagrams for the transducers measuring physical and mechanical parameters of fluids with specified functional and metrological characteristics. However, for a systematic solution of the task of choosing a throttle diagram and measuring channels of the transducer we need tools for modelling and analysis of diagram structures.

The purpose of the study is to develop the tools for structural modelling of throttle diagrams and choosing measuring channels using the graph theory, evaluating their number in order to synthesize transducers measuring physical and mechanical parameters and gas- or hydrodynamic characteristics of fluid with a specified functionality.

2. Classification of throttle measuring diagrams

In the previous papers [16, 17] we presented the following types of throttle diagrams: primary throttle element, primary cortege, primary rank, composite cortege, composite rank, throttle matrix. The primary throttle element (throttle) is a throttle diagram built on the basis of one throttle element. The primary cortege is a throttle diagram with a serial connection of primary throttle elements and the primary rank is a throttle diagram with a parallel connection of primary throttle elements. The number of elements of the primary cortege is called length and the number of elements of the primary rank is called width. It should be noted that a primary cortege of length 1 and a primary rank of width 1 transform into a primary throttle element. The composite cortege is a throttle diagram with a serial connection of primary throttle elements and primary ranks. The composite rank is a throttle diagram with a parallel connection of primary throttle elements and primary corteges.

A throttle matrix is investigated in [17]. It is a diagram containing parallel branches with the same number of in-series connected throttle elements. It should be noted that according to the developed description of throttle diagrams the throttle matrix is a composite rank with primary corteges of the same length.

However, not all throttle diagrams can be described with the existing definitions. Therefore, our task is to extend the existing concepts of composite ranks and composite corteges.

Let us divide a set of composite ranks into three groups: throttle matrices, partial throttle matrices and composite throttle matrices. The definition of a throttle matrix is discussed above. The definition of a partial throttle matrix is proposed as follows. It is a diagram containing parallel branches with different numbers of in-series connected throttle elements. Applying the terminology of throttle diagrams, a partial throttle matrix is a composite rank with primary corteges of different lengths (including length 1) as its elements.

A composite throttle matrix is the third type of composite ranks. We interpret it as a diagram which contains a parallel connection of primary corteges and branches with a serial connection of primary corteges with primary ranks, throttle matrices or partial throttle matrices. If one applies a composite throttle matrix instead of any throttle in the parallel branches of such a diagram it is possible to synthesize more complex structures of a composite throttle matrix.

We also propose to extend the definition of a composite cortege. It is a diagram with a serial connection of primary corteges, primary ranks and composite ranks. From the proposed definition it follows that a composite cortege must contain at least two types of diagrams, for example a primary cortege and a composite rank or a primary rank and a primary cortege etc. Consequently, a composite cortege is a throttle diagram that covers a lot of variants for constructing measuring diagrams.

Thus, taking into account the known and the proposed types of diagrams the authors developed a classification of throttle diagrams which is given in Table 1. The developed classification of throttle diagrams enables to investigate different diagrams for the purpose of constructing transducers for measuring fluid parameters.

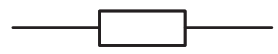
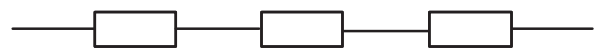
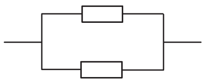
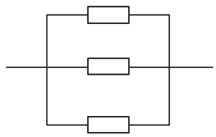
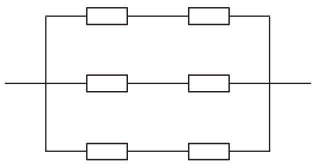
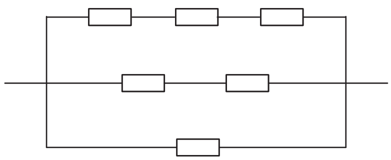
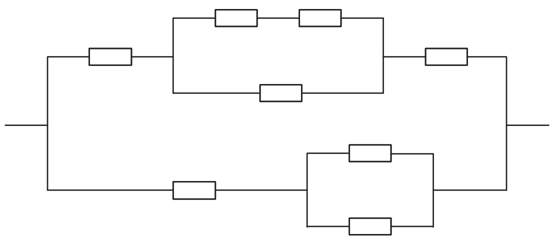
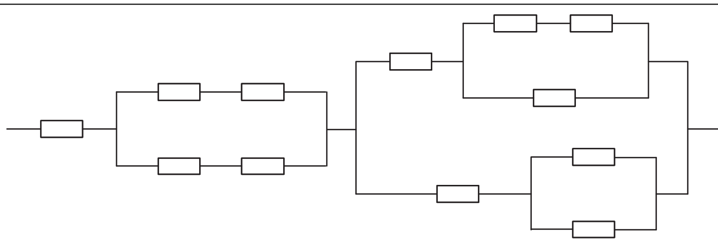
3. Modelling of throttle measuring diagrams

Let us apply the graph theory to modelling throttle measuring diagrams using the basic terms and definitions [18].

In order to perform a measurement the measuring channels are created in every throttle diagram. The connection points of throttle elements which are called diagram nodes are used for measuring channels. Such nodes of the diagram numbered in a certain way are the graph vertices and the measuring channels are the graph edges. Since the output signal of a measuring channel is the differential pressure between different nodes of the diagram, we can describe a measuring channel by a directionless edge connecting two different graph vertices. In this case, a non-oriented simple graph is obtained. Consequently, the graphs enable mathematical description of a set of nodes of the throttle matrix and a set of measuring channels. Taking into account transform functions of each measuring channel corresponding to the graph edges, we obtain a loaded graph which can serve as a basis for construction of mathematical models of measuring transducers.

It is necessary to mark the nodes in the diagram and to determine their number in order to describe a measuring diagram in the form of a graph. Let us analyse the throttle diagrams from Table 1 to determine the number of nodes in different types of diagrams.

Table 1. Classification of throttle diagrams.

No	Diagram name		Examples
1	Primary throttle element		
2	Primary cortege		 
3	Primary rank		 
4	Composite rank	Throttle matrix	
		Partial throttle matrix	
		Composite throttle matrix	
5	Composite cortege		

The simplest throttle diagram consists of one primary throttle. Obviously, the number of nodes in such a diagram equals 2. For a diagram with a serial connection of throttles, that is a primary cortege, the number n_c of nodes is proposed to be calculated by the formula:

$$n_c = n_{el} + 1, \tag{1}$$

where n_{el} is a number of primary throttle elements in cortege.

For example, Fig. 1a shows a primary cortege which contains 2 throttles and 3 nodes and Fig. 1b shows a primary cortege which contains 3 throttles and 4 nodes.

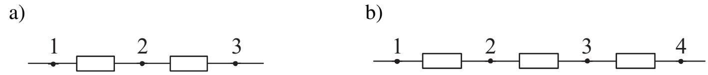


Fig. 1. Primary corteges: a) length 2; b) length 3.

Having analysed the throttle diagrams with a parallel connection of their elements we defined that the number of nodes in a primary rank does not depend on its width:

$$n_r = 2. \tag{2}$$

For example, Figs. 2a–2c shows that primary ranks of widths 2, 3, ..., p have 2 nodes regardless of the number of throttles in the diagram.

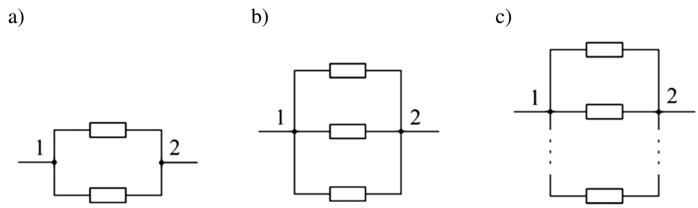


Fig. 2. Primary ranks: a) width 2; b) width 3; c) width p.

The simplest composite rank is a throttle matrix of size $p \times r$. It is investigated by the authors in [17], where the formula for determining the number n_m of nodes is proposed:

$$n_m = p \cdot (r - 1) + 2, \tag{3}$$

where p is the number of elements in each column of throttle matrix (the number of rows); r is the number of elements in each row of throttle matrix (the number of columns).

For example, Fig. 3 shows throttle matrices of various sizes with numbered nodes. Using (3) we find that the throttle matrix in Fig. 3a has 4 nodes, while the other two matrices (Figs. 3b, 3c) have 6 nodes each.

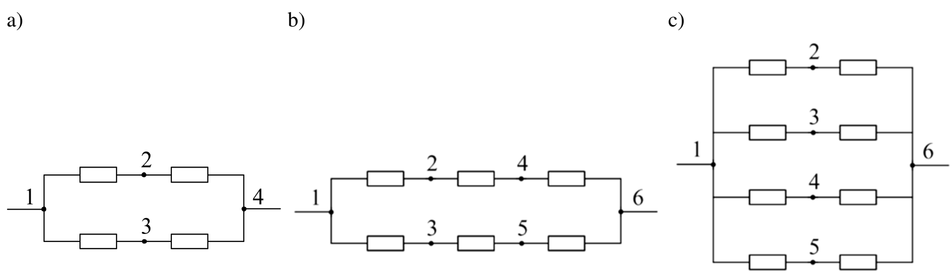


Fig. 3. Throttle matrices: a) size 2×2; b) size 2×3; c) size 4×2.

It is seen from Fig. 3 that the number of nodes in a throttle matrix increases with an increase in the number of throttle elements. For example, the matrix in Fig. 3a with 4 throttles has 4 nodes and the matrix in Fig. 3b with 6 throttles has 6 nodes. The matrices shown in Figs. 3b, 3c are of different sizes 2×3 and 4×2 with correspondingly different numbers (6 and 8) of throttle elements but they have the same number of nodes $n_m = 6$. Proceeding with (3), we can conclude that the number of nodes in throttle matrices with the same number of elements will be larger for the matrices with a larger number r of columns.

Let us consider the second type of composite rank – a partial throttle matrix. Such a diagram contains different number of in-series connected throttle elements in parallel branches. Taking into account (3) we propose a formula for determining the number n_{pm} of nodes in a partial throttle matrix:

$$n_{pm} = \sum_{i=1}^{p} (r_i - 1) + 2, \tag{4}$$

where p is the number of rows in the diagram; r_i is the number of throttle elements in i-th row of partial throttle matrix.

For example, Fig. 4 shows three diagrams of partial throttle matrices with marked nodes. The first diagram (Fig. 4a) contains 6 elements (3 elements – in the first row, 2 elements – in the second row, 1 element – in the third row). The second diagram (Fig. 4b) also contains 6 elements (4 elements – in the first row, 2 elements – in the second row) and the third diagram (Fig. 4c) contains 7 elements (2 elements – in the first and in the second rows, 3 elements – in the third row). Using (4) we determine that the first throttle diagram has 5 nodes while the other two diagrams have 6 nodes.

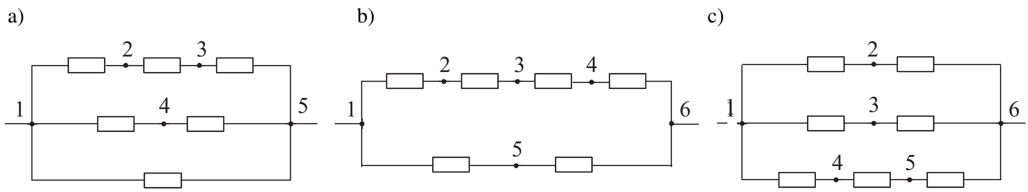


Fig. 4. Partial throttle matrices.

From Fig. 4 we see that the number of nodes in partial throttle matrices depends on the way of connection of throttle elements. Usually the number of nodes in a diagram increases with an increase in the number of elements (the diagram in Fig. 4a contains 6 throttles and 5 nodes while the diagram in Fig. 4c contains 7 throttles and 6 nodes). However, the diagrams with the same number of throttles but with different ways of their connection can have different numbers of nodes (each of the diagrams in Fig. 4a and Fig. 4b contains 6 throttles as well as 5 and 6 nodes respectively). Though the diagrams with different numbers of throttles can have the same number of nodes (the diagrams in Fig. 4b and Fig. 4c contain 6 throttles and 7 throttles respectively and both have 6 nodes).

Let us consider the third type of composite ranks, that is a composite throttle matrix. It is formed by a parallel connection of primary corteges and branches with a serial connection of primary corteges with primary ranks, throttle matrices or partial throttle matrices. We propose to calculate the number n_{cm} of nodes in a composite throttle matrix with the following formula:

$$n_{cm} = N_{pcm} + N_b + 2, \tag{5}$$

$$N_{pcm} = \begin{cases} 0, & \text{at } g = 0; \\ \sum_{i=1}^{g} (n_{el_i} - 1), & \text{at } g \geq 1; \end{cases} \tag{6}$$

$$N_b = \sum_{j=1}^{h} (n_{b_j} - 2), \tag{7}$$

where: N_{pcm} is the total number of nodes in the branches consisting only of primary corteges; g is the number of branches of primary corteges which are part of a composite matrix (if the composite matrix does not contain primary corteges, then $g = 0$); n_{el_i} is the number of elements in i-th primary cortege; N_b is the total number of nodes in the branches with a serial connection of primary corteges with primary ranks, throttle matrices or partial throttle matrices that are parts of a composite matrix; h is the number of such branches ($h \geq 1$); $g + h = p \geq 2$ is the total number of branches of composite matrix; n_{b_j} is the number of nodes in j-th branch which is calculated by the formula:

$$n_b = N_{pc} + N_{pr} + N_m + N_{pm} - (a + b + c + d - 1), \tag{8}$$

$$N_{pc} = \begin{cases} 0, & \text{at } a = 0; \\ \sum_{k=1}^{a} n_{c_k}, & \text{at } a \geq 1; \end{cases} \tag{9}$$

$$N_{pr} = \begin{cases} 0, & \text{at } b = 0; \\ \sum_{l=1}^{b} n_{r_l}, & \text{at } b \geq 1; \end{cases} \tag{10}$$

$$N_m = \begin{cases} 0, & \text{at } c = 0; \\ \sum_{r=1}^{c} n_{m_r}, & \text{at } c \geq 1; \end{cases} \tag{11}$$

$$N_{pm} = \begin{cases} 0, & \text{at } d = 0; \\ \sum_{s=1}^{d} n_{pm_s}, & \text{at } d \geq 1, \end{cases} \tag{12}$$

where: N_{pc} is the total number of nodes in primary corteges which are part of j-th branch of a composite matrix; a is the number of such primary corteges (if the j-th branch does not contain primary corteges, then $a = 0$); N_{pr} is the total number of nodes in primary ranks which are part of j-th branch of a composite matrix; b is the number of such primary ranks (if the j-th branch does not contain primary ranks, then $b = 0$); N_m is the total number of nodes in throttle matrices which are part of j-th branch of a composite matrix; c is the number of such throttle matrices (if the j-th branch does not contain throttle matrices, then $c = 0$); N_{pm} is the total number of nodes in partial throttle matrices which are part of j-th branch of a composite matrix; d is the number of such partial throttle matrices (if the j-th branch does not contain partial throttle matrices, then $d = 0$).

If one applies a composite throttle matrix instead of any throttle in a parallel branch of a composite throttle matrix, then the number of its nodes should be added into (8).

Figure 5 shows a diagram of a composite throttle matrix which is a parallel connection of two branches. For this diagram $N_{pcm} = 0$ and the total number of branches is $p = h = 2$.

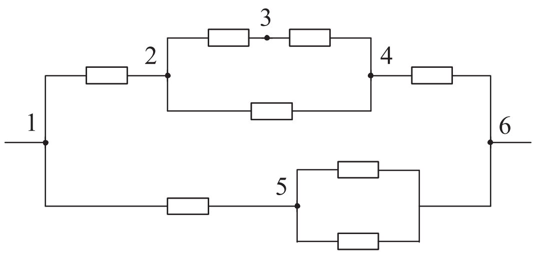


Fig. 5. A composite throttle matrix.

The first branch contains two corteges of length 1 and a partial throttle matrix, the second branch contains one cortege of length 1 and a primary rank of width 2. Taking into account that for the first branch $a = 2$ and $N_{pc} = 4$, $b = 0$ and $N_{pr} = 0$, $c = 0$ and $N_m = 0$, $d = 1$ and $N_{pm} = 3$ and using (8) we find that $n_b = 5$. For the second branch – $a = 1$ and $N_{pc} = 2$, $b = 1$ and $N_{pr} = 2$, $c = 0$ and $N_m = 0$, $d = 0$ and $N_{pm} = 0$ and $n_b = 3$, then using (7) we find the number $N_b = 4$. The total number of nodes is determined using (5) – $n_{cm} = 6$.

Finally, let us analyse diagrams of a composite cortege type. They can contain primary corteges, primary ranks and composite ranks (throttle matrices, partial throttle matrices and composite throttle matrices). With this in mind the authors proposed a formula for determining the number n of nodes in a composite cortege:

$$n = N_{pc} + N_{pr} + N_m + N_{pm} + N_{cm} - (a + b + c + d + e - 1), \tag{13}$$

where: N_{pc} is the total number of nodes in primary corteges which are part of a composite cortege, it is calculated by (9); a is the number of such primary corteges (if a composite cortege does not contain primary corteges, then $a = 0$); N_{pr} is the total number of nodes in primary ranks which are part of a composite cortege, it is calculated by (10); b is the number of such primary ranks (if a composite cortege does not contain primary ranks, then $b = 0$); N_m is the total number of nodes in throttle matrices which are part of a composite cortege, it is calculated by (11); c is the number of such throttle matrices (if a composite cortege does not contain throttle matrices, then $c = 0$); N_{pm} is the total number of nodes in partial throttle matrices which are part of a composite cortege, it is calculated by (12); d is the number of such partial throttle matrices (if a composite cortege does not contain partial throttle matrices, then $d = 0$); N_{cm} is the total number of nodes in the composite throttle matrices which are part of a composite cortege:

$$N_{cm} = \begin{cases} 0, & \text{at } e = 0; \\ \sum\limits_{i=1}^{e} n_{cm_i}, & \text{at } e \geq 1, \end{cases} \tag{14}$$

where e is the number of such composite throttle matrices (if a composite cortege does not contain composite throttle matrices, then $e = 0$). At least two of the values a, b, c, d, e should not be equal to 0 in order to obtain a composite cortege.

For example, let us consider a composite cortege containing 10 throttle elements and consisting of a primary element (cortege of length 1), a throttle matrix of size 2×2 and a partial throttle matrix of 5 elements (Fig. 6). Using (13) and taking into account the numbers of different types of diagrams ($a = 1$, $b = 0$, $c = 1$, $d = 1$, $e = 0$) we determine that this throttle diagram has 8 nodes.

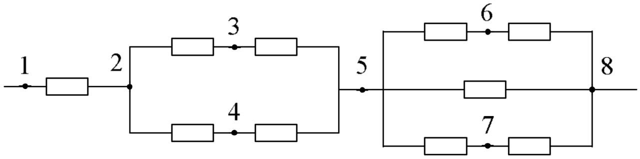


Fig. 6. A composite cortege.

It should be noted that a primary cortege or a primary rank, or a throttle matrix, or a partial throttle matrix, or a composite throttle matrix are partial cases of a composite cortege. So, the formula (13) can be used to calculate the number of n nodes in every throttle diagram.

It follows from the formulae (1)–(5), (8), (13) and Figs. 1–6 that the number of nodes in a diagram depends on its type and the way of connection of throttle elements.

Let us apply such terms of the graph theory as empty and complete graphs to describe throttle diagrams with measuring channels [18]. An empty graph describes a throttle diagram without measuring channels while a complete graph covers all possible measuring channels that can be built on the basis of a certain diagram. Therefore, the authors propose to determine the number m of all measuring channels that can be created in a certain throttle diagram with n nodes by the formula:

$$m = \frac{(n-1)\cdot n}{2}\,. \tag{15}$$

Usually all measuring channels of a throttle diagram are not involved simultaneously in measuring the physical and mechanical parameters of fluid. Such variants of construction of measuring transducers with one, two and several measuring channels should be described with the use of partial graphs. The set of all partial graphs of the complete graph covers all possible variants of construction of measuring transducers on a throttle diagram of a certain type which will differ in combinations of measuring channels.

The diagram with no measuring channels is described by an empty graph. With this in mind, based on the formula for calculating the number of partial subgraphs of a graph of size m [17] we derived the dependence used for determining the number n_{MT} of all variants of construction of measuring transducers on a certain throttle diagram with different numbers of measuring channels:

$$n_{MT} = 2^m - 1. \tag{16}$$

Let us consider some of throttle diagrams and their equivalent graphs (Fig. 7).

Thus, a primary cortege of length 3 (Fig. 7a) has 4 nodes in which 6 different measuring channels can be created (graph in Fig. 7b). A composite cortege consisting of a partial throttle matrix and a primary element (Fig. 7c) has 4 nodes in which 6 different measuring channels can be created (see graph in Fig. 7d). A partial throttle matrix consisting of two corteges of length 3 and length 1 (Fig. 7e) has 4 nodes in which 6 different measuring channels can be created (see graph in Fig. 7f). A composite throttle matrix (containing 6 elements) (Fig. 7c) consists of a primary cortege of length 2 and a composite cortege consisting of a primary element and a partial throttle matrix. It has 5 nodes in which 10 different measuring channels can be created (see graph in Fig. 7h).

From (15) and Fig. 7 we see that increasing of the number of diagram nodes leads to an increase in the number of measuring channels in it. As a result the number of transducers measuring the physical and mechanical parameters of fluids increases.

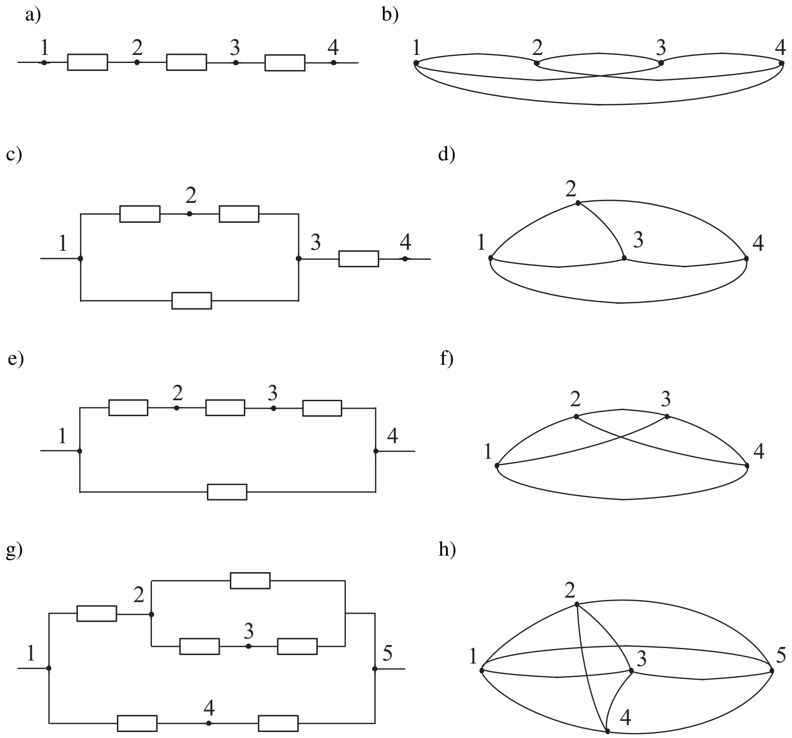


Fig. 7. Throttle diagrams and their equivalent graphs: a) a partial cortege of length 3; b) its equivalent graph; c) a composite cortege; d) its equivalent graph; e) a partial throttle matrix; f) its equivalent graph; g) a composite throttle matrix; h) its equivalent graph.

At the same time throttle diagrams with the same numbers of nodes have the same complete graphs (Fig. 7b, Fig. 7d, Fig. 7f). But, since they meet different types of throttle diagrams which may contain different throttles, the measuring transducers with different characteristics can be created on the basis of them. In order to analyse the possibilities of a certain diagram and its measuring channels, one should construct a model of this diagram in the form of a loaded graph with a set of edges and corresponding transform functions.

Based on our research we propose the following procedure for mathematical description of throttle diagrams using the graph theory:

1. Taking into account the fluid parameters to be measured one should select a throttle diagram. One should also take into account the number of parameters, the fluid properties, the requirements concerning accuracy, sensitivity of the transducer, *etc*.
2. Places of connection of throttle elements in the measuring diagram (diagram nodes), should be numbered and described in the form of vertices of the graph.
3. Each measuring channel of the throttle diagram should be described by a directionless edge which connects two different vertices of the graph. In this case, a non-oriented simple graph is obtained. The complete graph covers all measuring channels that can be created in a certain diagram.
4. Then, one should add transform functions to each measuring channel that correspond to certain graph edges. As a result one will obtain a loaded graph which is a generalized model of the throttle diagram with measuring channels.

5. After that one should analyse the possible partial loaded graphs. They correspond to the diagrams with only those measuring channels that will be used to measure specified fluid parameters.
6. As a result of the analysis one should choose a partial loaded graph and construct a mathematical model of the transducer measuring specified physical and mechanical parameters.

The developed procedure of mathematical description of throttle diagrams in the form of graphs makes it possible to construct generalized models of throttle diagrams of different types, to investigate functionality of the diagrams with different numbers of measuring channels in order to synthesize a mathematical model and a principle diagram of the transducer measuring specified fluid parameters.

4. Synthesizing measuring transducer based on composite throttle matrix using developed procedure

The developed procedure of mathematical description of throttle diagrams is applied to constructing a transducer measuring rheological parameters of non-Newtonian Bingham plastic fluid [19] on the basis of a throttle matrix. A schematic diagram of such a transducer is presented in Fig. 8. The cylindrical tubes of the same internal diameter are the sensitive elements of this transducer. The throttle elements are arranged in parallel branches, each of which consists of a serial connection of single tubes of different lengths and packets of tubes of different lengths. Branches differ only in the ways single tubes and packets are connected. The design feature of the throttles is the same difference of the lengths of single tubes and the lengths of packet tubes.

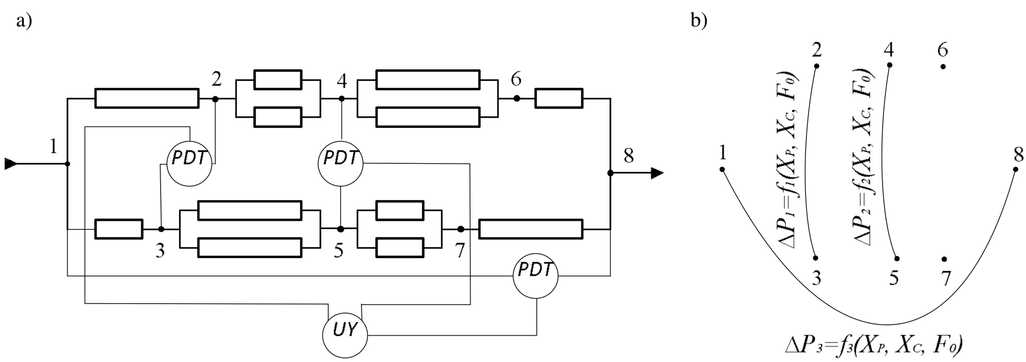


Fig. 8. A schematic diagram of a measuring transducer built on the basis of a composite throttle matrix (a) and its equivalent partial loaded graph (b): *PDT* – a differential pressure transducer; *UY* – a computing device.

A measuring transducer operates in the constant flow mode. Since the construction of each branch with throttles is identical, the hydraulic resistance of each branch will be the same. Consequently, the fluid flowrate through single tubes is twice less than the total flowrate F_0 of the controlled fluid which is fed into the throttle diagram. The flowrate in each of N packets tube is less than $2N$ times.

The differential pressures ΔP_1, ΔP_2 and ΔP_3 between the diagram nodes 2–3, 4–5 and 1–8 are the output signals of the transducer which are represented by the equivalent partial loaded graph (Fig. 8b).

The graph contains 8 vertices (vertex 6 and vertex 7 are isolated) and three loaded edges incident to vertices 1, 2, 3, 4, 5, 8 with their corresponding weight, namely the transform function, for each measuring channel:

$$\Delta P_i = f_i\left(X_P, X_C, F_0\right), \tag{17}$$

where: X_P is a vector of physical and mechanical parameters of the fluid; X_C is a vector of constructive characteristics of tubes and packet tubes; $i = 1,2,3$ is the number of measuring channel.

Consequently, the proposed transducer, measuring the plastic viscosity η, the yield stress τ_0 and the density ρ of Bingham plastic fluid, contains three measuring channels.

Let us determine a mathematical model of the transducer. The differential pressure that arises during motion of Bingham fluid in a single tube should be determined by the simplified Buckingham-Reiner equation [19, 20]:

$$\Delta P = \frac{64F_0L}{\pi D^4}\eta + \frac{16L}{3D}\tau_0 + \frac{2\alpha F_0^2}{\pi^2 D^4}\rho, \tag{18}$$

where: L is the length of a tube; D is the diameter of a tube; α is a correction coefficient for the input effects of tubes.

The differential pressure in each packet tube is determined by:

$$\Delta P = \frac{64F_0L}{N\pi D^4}\eta + \frac{16L}{3D}\tau_0 + \frac{2\alpha F_0^2}{N^2\pi^2 D^4}\rho, \tag{19}$$

where N is the number of tubes in a packet.

The first component of equations (18)–(19) depends on the viscous properties of fluid, the second component depends on the plastic properties of fluid. The third component is caused by the input effects of the tubes and depends on the density.

Let us determine the differential pressures between nodes 2–3, 4–5 and 1–8 in the measuring diagram. To do this let us determine the differential pressure between nodes 1 and 2:

$$P_1 - P_2 = \frac{64F_0L_{ls}}{\pi D^4}\eta + \frac{16L_{ls}}{3D}\tau_0 + \frac{2\alpha F_0^2}{\pi^2 D^4}\rho, \tag{20}$$

between nodes 1 and 4:

$$P_1 - P_4 = \frac{64F_0L_{ls}}{\pi D^4}\eta + \frac{16L_{ls}}{3D}\tau_0 + \frac{2\alpha F_0^2}{\pi^2 D^4}\rho + \frac{64F_0L_{sp}}{N\pi D^4}\eta + \frac{16L_{sp}}{3D}\tau_0 + \frac{2\alpha F_0^2}{N^2\pi^2 D^4}\rho, \tag{21}$$

between nodes 1 and 3:

$$P_1 - P_3 = \frac{64F_0L_{ss}}{\pi D^4}\eta + \frac{16L_{ss}}{3D}\tau_0 + \frac{2\alpha F_0^2}{\pi^2 D^4}\rho, \tag{22}$$

between nodes 1 and 5:

$$P_1 - P_5 = \frac{64F_0L_{ss}}{\pi D^4}\eta + \frac{16L_{ss}}{3D}\tau_0 + \frac{2\alpha F_0^2}{\pi^2 D^4}\rho + \frac{64F_0L_{lp}}{N\pi D^4}\eta + \frac{16L_{lp}}{3D}\tau_0 + \frac{2\alpha F_0^2}{N^2\pi^2 D^4}\rho, \tag{23}$$

and between nodes 1 and 8:

$$\Delta P_3 = P_1 - P_8 = \Delta P_{\eta,\tau_0} + \frac{4\alpha F_0^2}{\pi^2 D^4}\left(\frac{N^2+1}{N^2}\right)\rho, \tag{24}$$

where: L_{ls}, L_{ss} are lengths of long and short single tubes, respectively; L_{lp}, L_{sp} are lengths of tubes in long and short packets; $\Delta P_{\eta,\tau_0}$ is a component of the differential pressure in the tubes placed between nodes 1 and 8, which depends on the viscous and plastic fluid properties.

In order to determine the differential pressure between nodes 4 and 5 we subtract (23) from (21):

$$\Delta P_2 = P_5 - P_4 = \frac{N-1}{N} \cdot \frac{64F_0\Delta L}{\pi D^4}\eta, \tag{25}$$

where $\Delta L = (L_{ls} - L_{ss}) = \left(L_{lp} - L_{sp}\right)$ is the difference between lengths of long and short single tubes and packets. So the constructive characteristics of the tubes and the channel between nodes 4 and 5 are chosen in order to compensate the impact of the yield stress and density on the plastic viscosity measurement.

In order to determine the differential pressure between nodes 2 and 3 we subtract (22) from (20):

$$\Delta P_1 = P_3 - P_2 = \frac{64F_0\Delta L}{\pi D^4}\eta + \frac{16\Delta L}{3D}\tau_0 . \tag{26}$$

Applying the formulae (24), (25) and (26) we obtain a static mathematical model of the proposed measuring transducer:

$$\begin{cases} \eta = k_1 k_2 \Delta P_2; \\ \tau_0 = k_3 \left(\Delta P_1 - k_1 \Delta P_2\right); \\ \rho = k_4 k_5 \left(\Delta P_3 - \Delta P_{\eta,\tau_0}\right). \end{cases} \tag{27}$$

where $k_1 = \frac{N}{N-1}$, $k_2 = \frac{\pi D^4}{64F_0\Delta L}$, $k_3 = \frac{3D}{16\Delta L}$, $k_4 = \frac{\pi^2 D^4}{4\alpha F_0^2}$, $k_5 = \frac{N^2}{1+N^2}$ are proportionality coefficients. The computing device *UY* calculates the values of fluid parameters by using the differential pressures ΔP_1, ΔP_2 and ΔP_3 measured by differential pressure transducers.

Thus, using the developed procedure for a mathematical description of throttle diagrams we constructed a model in the form of a partial loaded graph (Fig. 8b), a schematic diagram of transducer measuring rheological parameters of Bingham plastic fluid (Fig. 8a) and its mathematical model (27). The obtained mathematical model was validated by simulation for Newtonian and Non-Newtonian fluids with viscosity of 5–50 mPa·s, yield stress of 0–10 Pa and density of 1000–1400 kg/m^3 using tubes with a diameter of 2–6 mm, a length difference of 0.1–0.3 m and a flowrate of 60–120 L/h. The results of the simulation proved the possibility of hardware creation of a measuring transducer on the basis of the proposed diagram.

5. Conclusions

In the paper the problem of determining the possible variants of constructing measuring transducers on the basis of throttle diagrams of different types was solved using the graph theory and combinatorics.

A classification of throttle diagrams was developed which includes primary elements, primary corteges, primary ranks, composite corteges and composite ranks. New definitions of a partial throttle matrix and a composite throttle matrix were proposed. The definitions of a composite cortege and composite ranks have been improved. Using the combinatorics tools the dependences were obtained for determining the number of nodes and the number of measuring channels in throttle diagrams of different types, as well as the number of possible variants of measuring transducers constructed on the basis of a certain diagram.

A procedure for mathematical description of throttle diagrams was developed which makes it possible to construct mathematical models of throttle diagrams of different types in the form of graphs, to investigate functionality of diagrams with different numbers and arrangements of measuring channels. The model was developed in the form of a graph, a schematic diagram and a mathematical model of the transducer measuring physical and mechanical parameters of a Bingham plastic fluid, namely its plastic viscosity, yield stress and density, was developed based on a throttle matrix.

The obtained results make it possible to synthesize a hydro-gas-dynamic throttle transducer measuring one or several fluid parameters with specified functionality and metrological characteristics. The developed procedure and dependences will be useful for creating algorithms of the process of synthesizing diagrams of transducers measuring specified fluid parameters. This is the subject of the further research.

References

[1] Webster, J.G., Eren, H. (2014). *Measurement, Instrumentation, and Sensors Handbook: Spatial, Mechanical, Thermal, and Radiation Measurement*. CRC Press Reference.

[2] Profos, P., Pfeifer, T. (1994). *Handbuch der industriellen Meßtechnik*. Oldenbourg Wissenschaftsverlag.

[3] Clarke, P.G., Murphy, M.P. (2017). U.S. Patent 9,612,183 B2. Modular Capillary Bridge Viscometer.

[4] Drevetskiy, V., Klepach, M. (2013). The Intelligent System for Automotive Fuels Quality Definition. *Informatics, Control, Measurement in Economy and Environment Protection*, 3(3), 11–13.

[5] Kutia, V.M. (2015). Automated System for Continuous Viscosity Measurement of Emulsions. *ICACIT-2015: Proc. of the 3rd International Conference on Automatic Control and Information Technologies*, 124–127.

[6] Nicholls, M., Murphy, M.P. (2014). U.S. Patent Appl. 2014/014.4214. Balanced Capillary Bridge Viscometer.

[7] Trainoff, S.P. (2008). U.S. Patent 7,331,218 B2. Capillary Bridge Viscometer and Method for Measuring Specific Viscosity.

[8] De Corral, J.L. (1997). U.S. Patent 5,637,790. Three Capillary Flow-Through Viscometer.

[9] Van Oort, E., Vajargah, K., Hoxha, B. (2015). U.S. Patent Appl. 2015/0330213 A1. Systems and Methods for Determining a Rheological Parameter.

[10] Stasiuk, I. (2015). Gas Dynamical Capillary Flowmeters of Small and Micro Flowrates of Gases. *Energy Engineering and Control Systems*, 1(2), 117–126.

[11] Pistun, E.P., Teplyukh, Z.N., Dilai, I.V., Drul, Ya. G. (1990). Construction of Measuring Converters of Small and Micro Gas Flows in Linear Gas-Dynamic Throttles. *Measurement Techniques*, 33(8), 809–811.

[12] Matiko, H., Pistun, Y. (2015). Gas-dynamic Analyzer of Nitrogen-hydrogen Mixture for Industrial Application. *Energy Engineering and Control Systems*, 1(2), 101–110.

[13] Pistun, E., Standa, J. (2006). *Pomiary ilości oraz strumienia masy i objętości przepływających płynów*. Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław.

[14] ISO 5167-1:2003. Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full – Part 1: General principles and requirements.

[15] Pistun, Ye., Lesovoy, L., Fedoryshyn, R., Matiko, F. (2014). Computer Aided Design of Differential Pressure Flow Meters. *World Journal of Engineering and Technology*, 2(2), 68–77.

[16] Pistun, Ye., Matiko, H., Krykh, H. (2016). Modeling the schemes of hydrodynamic measuring transducers using set theory. *Metrology and Measurements*, 3, 53–61.

[17] Pistun, Ye., Matiko, H., Krykh, H., Matiko, F. (2017). Synthesizing the schemes of multifunctional measuring transducers of the fluid parameters. *Eastern-European Journal of Enterprise Technologies*, 6, 5(90), 13–22.

[18] Diestel, R. (2016). *Graph Theory*. Springer-Verlag, Heidelberg.

[19] Rao, M.A. (2014). *Rheology of Fluid, Semisolid, and Solid Foods, Principles and Application*. Food Engineering Series, Springer Science + Business Media, New York.

[20] Steffe, J.F. (1996). *Rheological methods in food process engineering*, USA, Freeman Press.