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TEKA. COMMISSION OF MOTORIZATION AND ENERGETICS IN AGRICULTURE – 2016, Vol. 16, No.3, 21-30

Research of heat energy transfer processes based on rheological transitions theory and zero gradient method

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Received August 05.2016: accepted August 08.2016

Summary. It is shown that heat energy transfer from the source to the medium is accompanied by rheological transitions. Physical parameters of the medium change in the rheological transition zone due to heat energy flow transfer at a certain speed. It is shown that use of linear gradient laws during description of heat energy transfer processes leads to great differences between theoretical and experimental results, as well as the paradox of infinite spreading speed of disturbances of temperature fields. For mathematical description of heat energy transfer processes in mediums, it is proposed to use the method of irreversible rheological transitions and zero gradient, thus providing solutions of nonlinear differential equations in analytical form.
Key words: rheological transition; multiphase environment; heat transfer; modeling.

INTRODUCTION

All the physical and chemical processes run at a certain temperature which exceeds thermodynamic zero minus 273.15 K. A temperature of 293.15 K or 20°C is accepted as normal. Temperature has significant impact on almost all the processes of human life. Transfer of mass and momentum are also closely connected with the heat energy transfer process. At a temperature above absolute zero, heat energy flow transfer in the medium takes place due to heat conductivity and convection. Molecular diffusion corresponds to heat transfer by molecular heat conductivity; convection diffusion corresponds to heat transfer by convection heat conductivity. Experimental study of heat energy flow transfer is complicated by the need to make measurements in the medium with variable temperatures. Thus, the results are affected by dependence of physical constants on temperature. So, one has to use values of these constants mediated by temperature, and the results of experimental data processing depend on the mediation way. The most accurate data to calculate convection heat exchange processes are obtained by the method of analogy with diffusion. For stationary medium, the fundamental law of heat energy flow transfer is Fourier's law, according to which the heat flow is proportional to the temperature gradient [1]:

$$q = -\lambda \,\mathrm{grad} T \equiv -\lambda \frac{dT}{dy}, \qquad (1)$$

where: q is the heat energy flow transferred through surface unit per time unit, $\mathrm{grad} T$ is the gradient of temperature T change throughout linear coordinates directed along the normal to the surface, through which heat energy transfer takes place, λ is the heat conductivity coefficient.

The minus sign indicates that heat energy transfer takes place in the direction, in which temperature decreases, i.e. in the direction of the negative temperature gradient. Fourier's law in form (1) describes heat energy flow transfer in the homogeneous chemical composition medium. Research of heat energy transfer processes is generally based on the fact that isothermal distribution of heat energy takes place in some volume. For non-isothermal diffusion in the gaseous medium, the gradient is replaced by the partial pressure gradient $p = RTC$, and the diffusion coefficient – by the value D/RT, where R is the gas constant, T is absolute temperature. Describing heat energy transfer processes to the medium characterized by convective properties, Fourier's law is supplemented by the components, reflecting convective transfer of the heat energy flow. If the linear speed of this flow is marked by v, Fourier's law takes the form of:

$$q = -\lambda \,\mathrm{grad}\, T + c_P \rho v T, \qquad (2)$$

where: c_P is heat capacity at constant pressure, ρ is thickness (or density).

For heat energy flow transfer processes, there is used the so-called heat conductivity coefficient a, which is connected with the conventional heat conductivity coefficient by the ratio $a = \lambda / c_P \rho$.

LITERATURE DATA ANALYSIS AND PROBLEM DEFINITION

In recent years, the issue of heat energy transfer processes concerned only applied nature. Basically, there were researched the problems of impact of heat on substances' conversion processes in chemical technologies [2], work of dimensional control means [3-7], work of machines and mechanisms [8]. Papers generally consider unilateral processes of heat energy transfer, during which it is assumed that heat energy sources has infinite power and transfer speed. Such processes are described by quite complex integral-differential equations, based on the known Fourier's law of heat energy transfer. Typically, such equations are nonlinear, and they have no analytical solution, so they are reduced to simpler forms that can be taken as linear.

Basically, there were studied processes of heat energy transfer from the source to the medium without considering the peculiarities of transfer on the interface. For the first time this fault was highlighted by professors Weisberg M.A. [9], Taganov I.M. [10] and Gorazdovs'kij T. Ja. [11]. In his works, Prof. Gorazdovs'kij T. Ja. firstly discovered thermorheological effect, the essence of which is that on the interface of two fluid (liquid, gaseous or viscoplastic phases), there are phenomena of wall-adjacent thermophysical nature, affecting not only technological processes but also metrological incorrectness at experimental researches of rheological properties of substances and materials, as well as wrong measurement results of different technological processes. In addition, wrong theoretical generalization leads to wrong recommendations in technologies, large losses of material and energy resources. As indicated in [9], in the thermally isolated medium, a source of heat can be dissipative energy, which is released within fluid (gas, liquid, solid body, etc.) due to viscous friction or destruction of the inner frame of the substance exposed to force-torque (rheological) voltages of different levels relative to values of critical voltages of phase rheological transitions peculiar to the given fluid. Such effects, which are caused by energy dissipation in the rheological transition zone (wall-adjacent zone) of substances with different rheological and thermophysical properties, are important and urgent for the thermodynamics theory [12] as well as metrology and practice.

Temperature is usually associated with transfer of the heat energy flow from its source to the medium. As a result of this transfer, its conversion into other forms of energy takes place, e.g. into molecular motion during friction, in liquid and gaseous mediums. In many cases, heat energy transfer is accompanied by chemical conversions that create new sources of heat energy [13, 14]. In many cases, there are used differential equations that describe heat transfer processes. The heat conductivity equation in the stationary medium is:

$$c_P \rho \frac{\partial T}{\partial \theta} = \operatorname{div} \lambda \cdot \operatorname{grad} T + q', \tag{3}$$

where: q' is the heat energy flow of the source, which appears, e.g., as a result of a chemical reaction.

Equation (3) is true in the case of considering the process of heat energy transfer in the medium determined by appearance of the flow q'. On the other hand, appearance of such a flow requires disturbance in this medium with infinite volume speed. Considering the heat balance equation for certain enclosed volume of the single-dimensional temperature field, it can be assumed that the amount of heat energy transferred to the medium per time θ equals to the amount of heat energy q_x transferred throughout the linear coordinate *x*. That is:

$$c_P \rho \frac{\partial T}{\partial \theta} = -\frac{\partial q_x}{\partial x}. \tag{4}$$

Since the amount of heat energy is:

$$q_x = -\lambda \frac{\partial T}{\partial x} - \tau_P \frac{\partial q_x}{\partial \theta}, \tag{5}$$

where: τ_P is the time constant of the heat transfer process, then applying (5) to equation (4), we get:

$$\tau_P \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial T}{\partial \theta} = a \frac{\partial^2 T}{\partial x^2}, \tag{6}$$

where: $\tau_P = a / v^2$ is the time constant of the process of heat energy transfer from the internal source, $a = \lambda / c_P \rho$ is the temperature conductivity coefficient.

Since, according to the problem condition, heat transfer is limited by some enclosed volume, beyond which heat energy does not spread, then the temperature of this medium may vary from the initial *Tn* to the final *Tк*, which is created by the heat source q_x. However, in this case, the source cannot be of infinite power. Such boundary conditions lead to the fact that descript-ion of processes of heat energy transfer is approximate. Among a large number of problems there are usually considered only major classical problems, for which it is characteristic that corresponding boundary conditions (initial and final) are set that allow finding solutions in the simplest analytic form. Under boundary conditions of the first kind, the surface temperature is set as a function of time. The literature given below considers the simplest cases, when the surface temperature of the body remains constant throughout the heat transfer process. It corresponds to the case, when the heat source power is infinite, and all its heat transfers to the heating medium. This case corresponds to the problem when the transient process of heat exchange is defined when the heated object is exposed to the single step signal (the heat amount). Boundary conditions of the second and the third kinds are supplemented by additional conditions that are imposed upon the differential equation of heat energy transfer. Typically, the heat conductivity research is limited by geometrical and physical parameters of the studied object. In many cases, it is assumed that the speed of heat energy distribution is infinitely great. In this case, the time constant $\tau_P = 0$ and equation (6) are presented as:

$$\frac{\partial T(x,\theta)}{\partial \theta} = a \frac{\partial^2 T(x,\theta)}{\partial x^2}. \tag{7}$$

While solving this equation, the following conditions are imposed: time $t > 0$; direction change $-\infty < x < +\infty$; at the initial time $t = 0$ temperature distribution is given, that is $T(x,0) = f(x)$; as the body size is unlimited, then $\frac{\partial T(+\infty,\theta)}{\partial x} = \frac{\partial T(-\infty,\theta)}{\partial x} = 0$. The problem solution can be obtained by Fourier's method, but, at the same time, certain limitations associated with the possibility of representing the function $f(x)$ as a Fourier's integral will be imposed upon the function $f(x)$. For this purpose, it is enough to have the integral $\int_{-\infty}^{+\infty} \left| f(x) \right|^2 dx$.

Therefore, the solution by the method of sources, which does not require limitations for the function $f(x)$, is given. In [15-17] it is indicated that partial solution of equation (7) is the following one:

$$T(x,\theta)=\frac{C}{\sqrt{4\pi a\theta}}\exp\left[-\frac{(x-\xi)^2}{4a\theta}\right], \qquad (8)$$

where: C is a constant, ξ is the current constant of the direction x.

If temperature is $T(0,\theta)=T_C=const$, where: T_C is the temperature of the medium, then by introducing a new variable $\vartheta=T(x,\theta)-T_C$, the equation of heat energy transfer in the direction x will be:

$$\frac{T(x,\theta)-T_C}{T_0-T_C}=\operatorname{erf}\left(\frac{x}{2\sqrt{a\theta}}\right). \qquad (9),$$

where: T_0 is the temperature of the heat energy source.

The problem of research of a semibounded body without thermal insulation of the lateral surface belongs to one of classic. Between the lateral surface of the body and the medium, heat exchange takes place by Newton's law. Temperature of the medium that limits the lateral surface of the body is taken as constant and equal to its initial temperature. If the height and the width of the body are quite small compared with the length and the heat conductivity coefficient is significant, it can be considered that the temperature drop throughout the height and the width of the body is constant. That is, temperature change derivatives throughout other linear coordinates, for example, y i z are zero. Thus, this problem is reduced to one-dimensional when the temperature drop is only in one direction x. Heat transfer from the lateral surface of the body to the medium is taken into account only in the differential equation itself as the negative heat source. Thus, the differential equation of heat conductivity is written as follows:

$$c_P\rho\frac{\partial T(x,\theta)}{\partial\theta}=\lambda\frac{\partial^2T(x,\theta)}{\partial x^2}-q^w. \qquad (10)$$

where: q^w is the heat amount given by the body volume unit per time unit to the medium.

The following conditions are taken as boundary: when $\theta>0$ and $0<x<\infty$ $q^w=(\alpha/h)[T(x,\theta)-T_0]$, where α is the heat exchange coefficient; $h=S/\mathrm{P}$; S is cross-sectional area of the rod; P is perimeter of the body. Then equation (10) is reduced to the following form:

$$\frac{\partial T(x,\theta)}{\partial\theta}=a\frac{\partial^2T(x,\theta)}{\partial x^2}-\frac{\alpha}{c_P\rho h}[T(x,\theta)-T_0]. \qquad (11)$$

Since solution of equation (11) is complex, then it is taken for simplification that the ratio of the end of the body is $\alpha/\lambda=\infty$. This means that the temperature of the end of the rod becomes constant and equal to the temperature of the medium T_C at once. Then for boundary conditions it can be written: $T(x,0)=T_0$, $T(0,t)=T_C$, $T(\infty,t)=T_0$ and $\frac{\partial T(\infty,t)}{\partial x}=0$. With such a simplification, the solution of equation (11) takes the following form:

$$\frac{T(x,t)-T_0}{T_C-T_0}=\frac{1}{2}\left\{\left[\exp\left(-x\sqrt{\frac{\alpha}{\lambda h}}\right)\right]\operatorname{erfc}\left(\frac{x}{2\sqrt{a\theta}}-\sqrt{\frac{\alpha a\theta}{\lambda h}}\right)+\right.$$
$$\left.+\left[\exp\left(x\sqrt{\frac{\alpha}{\lambda h}}\right)\right]\operatorname{erfc}\left(\frac{x}{2\sqrt{a\theta}}+\sqrt{\frac{\alpha a\theta}{\lambda h}}\right)\right\} \qquad (12)$$

where: h is the length of the body.

As it is seen from the above, the problems of heat energy transfer from its source to the medium are quite complex. Analysis of the literature given below shows that there are generally considered unilateral simplest equations of linear type with different initial and boundary conditions. The basic is Fourier's equation (1) of heat energy transfer by heat conductivity. There are considered various options for heat flow transfer, but in almost all the cases it is assumed that this transfer is carried out along the linear coordinate that varies from 0 to ∞, the source of heat energy is infinite, as its temperature on the transfer boundary (at $x = 0$) is constant, and boundary conditions should be as follows: $T(x,0)=T_0$, $T(0,\theta)=T_C$, $T(\infty,\theta)=T_0$ and $\partial T(\infty,\theta)/\partial x=0$. Such boundary conditions that allow finding solutions of differential equations of heat energy transfer, as pointed out by Prof. Weinberg A.M. in [9], lead to the paradox, since they specify that the transfer speed should be infinite. On the other hand, the problems of heat energy transfer are unilateral, as energy is transferred from the source of infinite power throughout certain coordinates into infinity, violating the physical nature, as all the physical bodies are limited in their size and characterized by the corresponding variable volume. If the volume is infinite, then infinite amount of heat energy can be transferred into it. Analytical equation (8) for a perfectly isolated body and equation (11) for a body with a bare butt are quite complex, especially for their practical use in modern computer-integrated systems of control and industrial processes management. Moreover, energy conservation law is not provided for heat energy transfer in volumes of real mediums under above conditions. All the transfer processes of energy, mass and momentum in physical mediums involve irreversible rheological transitions. These processes have appropriate boundaries (rheological transitions zones), which can be both infinitely small and infinitely large. In these zones conversion of heat energy takes place (molecules, atoms, ions, etc. begin moving to the medium into which heat is transferred). Almost all the processes in chemical technology involving heating or cooling of substances, as well as temperature errors of dimensional control in information control systems take place due to heat energy transfer. Accordingly, the problems of mathematical description of heat energy transfer processes in different mediums ensuring adequacy of calculations to experimental results is relevant.

PURPOSE AND OBJECTIVES OF RESEARCH

The purpose is to research heat energy transfer processes based on the theory of irreversible rheological transitions and principles for development of mathematical models for bounded and semibounded mediums.

To achieve this purpose it is necessary to solve the following problems:

- to develop physical models of irreversible rheological transitions (IRT) for heat energy transfer processes;
- to describe IRT with the help of nonlinear differential equations of heat energy transfer with dissipative function of the flow speed;
- to find the analytic solution of nonlinear differential equations of the heat energy transfer speed based on the zero gradient method.

GENERAL FORMULATION OF RESEARCH OF HEAT ENERGY TRANSFER BY RHEOLOGICAL TRANSITIONS METHOD

Mathematical description of nonlinear processes of heat energy transfer is highlighted in a significant number of scientific papers below. They firstly describe heat and mass transfer processes, when there are deviations from linear Fourier's and Fick's laws. Nonlinear generalization of Newton's law in the theory of momentum impulse transfer led to development of the theory of rheological transitions and conversions that gave rise to present Fourier's law of heat conductivity in the more generalized ratio of the following type:

$$q_T = -\int_0^{\infty} k(\xi)\cdot g_T(\theta-\xi)d\xi\,, \qquad (13)$$

where: q_T is the heat energy flow; $k(\xi)$ is a function describing rheological transition; g_T is the temperature gradient, ξ is time of rheological conversion of heat energy; θ is time of transfer of the heat energy flow to the other medium.

If we compare equation (13) with the description of integrated impulse Dirac delta function [15]:

$$\int_{t_1+0_1}^{t_2} f(\xi)\delta(\theta-\xi)d\xi = \begin{cases} 0 & \theta_0<\theta_1,\quad \theta_1\ge\theta_2 \\ f(\theta_1+0) & \theta_1<\theta_0<\theta_2 \end{cases}, \qquad (14)$$

where: ξ is a variable, $f(\xi)$ is the function describing the heat energy transfer process with heredity, $\delta(\theta-\xi)$ is the core of linear integral conversion, θ_0 is average time of phase transfer, θ_1, θ_2 timeframes of integrated impulse Dirac delta function, then one can note their similarities.

Firstly, if in equation (13) integration limits vary from zero to infinity, then in (14) they vary within some real limits: from θ_1 to θ_2. Secondly, a function is a particular function describing the heat energy transfer process with heredity. Thirdly, the temperature gradient $g_T(\theta-\xi)=\delta(\theta-\xi)$ is the core of linear integral conversion if time is $\theta=\xi$. Since this process is characterized by speed of heat energy transfer, equation (14) can be written in the following form:

$$\int_{x_1+0_1}^{x_2} f(\zeta)\delta(x-\zeta)d\zeta = \begin{cases} 0 & x_0<x_1,\quad x_1\ge x_2 \\ f(x_1+0) & x_1<x_0<x_2 \end{cases}, \qquad (15)$$

where: ζ is a linear coordinate variable, $f(\zeta)$ is the function describing the heat energy transfer process with heredity to distance ζ, $\delta(\zeta-x_0)$ is the core of linear integral conversion, x_0 is average time of phase transfer, x_1, x_2 are linear limits of integrated impulse Dirac delta function.

According to equations (14) and (15), the process of heat energy transfer can be considered both regarding time and a geometrical coordinate corresponding to Fourier's law. Fig. 1, 2 show a physical model of irreversible rheological transition of the heat energy flow from the source to the medium (Fig. 1) and a diagram of integrated impulse Dirac delta function (Fig. 2) in the form of a rectangle, within which there are diagrams of heat energy (temperature) transfer from the source to the medium (curve 1), and the curve of this energy storage in the medium (curve 2). The diagram of integrated impulse Dirac delta function is limited on the left by the straight *a-b*, and on the right – by the straight *c-d*. Thus, at the border *a-b* (point θ_1 (x_1)) and *c-d* (point θ_2 (x_2)), gradients of temperature change are $\frac{\partial T_{a-b}}{\partial\theta}=0$ and $\frac{\partial T_{c-d}}{\partial\theta}=0$. The zone, bounded by the rectangle *a-b-c-d*, in which the process of heat energy conversion takes place, is called as the irreversible rheological transition (IRT) zone.

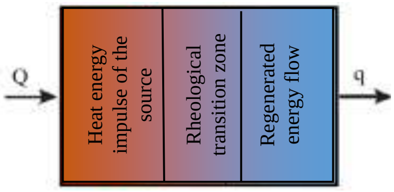

Fig. 1. Physical model of the process of heat energy impulse transfer

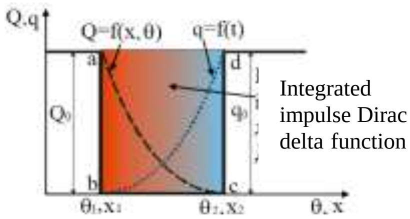

Fig. 2. IRT diagrams and integrated impulse Dirac delta function

Since within the integrated impulse Dirac delta function there are normally two IRT curves – the curve of change in energy transfer $T_{11}=f(x,\theta)$ and the curve of the conversion process flow $T_{22} = f(t)$, then the heat energy balance is always executed, i.e.:

$$\bar{T}(\bar{x},\theta) = \bar{\varphi}(\bar{x},\theta)\cdot\bar{v}(\bar{x},\theta) + \bar{f}(t), \tag{25}$$

where: $\bar{\varphi}(\bar{x},\theta)$ is the potential transfer vector, $\bar{x}$ is the vector of transfer motion direction, $\bar{v}(\bar{x},\theta)$ is the vector of the transfer speed, t is the transfer time of the result of the heat energy flow conversion from the IRT zone to the other volume, where it accumulates.

Since in the IRT zone, the temperature $T_{11}(x,\theta)$ ranges from T_{00} to T_{10} according to Fourier's law, then according to [3, 4] equation (9) is written as follows:

$$\frac{T_{11}(x,\theta) - T_{10}}{T_{00} - T_{10}} = \mathrm{erf}\left(\frac{x}{2\sqrt{a\cdot\theta}}\right), \tag{26}$$

where: a is thermal conductivity, T_{00} is the temperature of the heat flow, which is transferred from the source to the IRT zone.

Equation (26) is correct when the heat energy transfer time is limited to a certain value θ_{10} at which the body changes its temperature from the initial T_{00} to T_{10}. But it is necessary to terminate exposure of the body to the source. If heat transfer takes place from $\theta = 0$ to $\theta = \infty$, then equation (26) takes the following form:

$$T_{11}(x,\theta) = T_{00}\mathrm{erf}\left(\frac{x}{2\sqrt{a\cdot\theta}}\right). \tag{27}$$

As it is shown in physical model of Fig 1, *a*, in the simplest case the physical model of the process of heat energy transfer consists of three stages. At the first stage, the impulse of heat energy *Q* moving to the other medium creates an impulse for rheological transition. Such an impulse can be thickness of the laminar layer near the heating wall, phases' interface, friction place between two bodies, chemical reactor, etc. This impulse is formed in a certain area (rheological transition zone), in which the heat impulse from the source is converted either into heat energy of the other medium or mechanical movement, or substance mass transfer, or chemical reaction. During this conversion of heat energy of the source, new energy (e.g., thermal, mechanical, chemical or other) appears, which always comes from the IRT zone (flows) at a certain speed and accumulates (integrates) in this medium. If the newly created energy does not come from the IRT zone, it will accumulate there until it fills the entire volume of the zone. Having reached such a state, the heat energy transfer process stops. An example of such transfer is the heating process of the body with the perfectly insulated outer surface. When the temperature of the heating body reaches the temperature of the incoming heat energy flow, then the heat transfer process eliminates. On the other hand, when it is assumed that power of the heat energy source is infinite, it leads, for example, to change of the phase state of the isolated medium (conversion of the solid medium into liquid or gaseous), at which the heat energy transfer process stops. These processes run in accordance with relevant laws (Fourier's law for heat energy impulse transfer, Fick's law for substance mass impulse transfer, Newton's law for momentum impulse transfer), which are combined by the efficient transfer coefficient –thermal conductivity, mass conductivity and kinematic viscosity correspondingly. All these factors have the same dimension (m^2/s). For the process of heat energy (temperature) flow transfer in the body with the perfectly insulated surface, as mentioned above, Fourier's law in the following form is used:

$$v = \frac{\partial T(x,\theta)}{\partial\theta} = -a\frac{\partial^2 T(x,\theta)}{\partial x^2}. \tag{27}$$

Accumulation of heat energy in the IRT zone takes place according to formula (26). Equation (27) describes speed v of accumulation of heat energy in the body, which is also the IRT zone. If $\partial x^2 = v^2\partial\theta^2$ then equation (27) is reduced to the following:

$$\frac{\partial}{\partial\theta}\left(\frac{a}{v^2}\frac{\partial T(\theta)}{\partial\theta} + T(\theta)\right) = \frac{\partial}{\partial\theta}(x). \tag{28}$$

Analysis of equation (28) shows that when $\partial x^2 = v^2\partial\theta^2$, then the heat energy transfer process is described by the linear differential equation of the first order:

$$\tau_\theta\frac{\partial T(\theta)}{\partial\theta} + T(\theta) = k_\theta L, \tag{29}$$

where: $\tau_\theta = a/v^2$ is the constant of heat energy transfer time from the source to the IRT zone, k_θ is the transfer coefficient, L is the total distance of the direction of the heat energy flow distribution.

By analogy with (26) solution of equation (29) during transferring of heat energy from $\theta = 0$ to $\theta = \theta_{10}$, we get:

$$T(\theta) = k_\theta L\exp(-\theta/\tau_\theta), \tag{30}$$

If the result of conversion of the heat energy flow is being continuously deduced from the IRT zone (e.g. a semibounded body) to the different medium, the latter is cumulative. Accumulation takes place at a certain speed, which is described by the dissipative function $f(t)$, where: t is the flow time of the conversion result. The conversion result can be both heat energy (e.g., heating of the medium by electric current) and substance weight (e.g., substance dissolving, evaporation, etc.) or new substances generated in the result of chemical reactions. Theoretical and experimental studies show that the flow speed of the determining parameter R, which may be the temperature of the heat flow or solution concentration, or momentum, can be described by the following linear differential equations:

- Integrating dynamic element: $f(t) = \dfrac{dR}{dt}$;

- Aperiodic dynamic element of the first order: $f(t) = \tau_1\dfrac{d^2R}{dt^2} + \dfrac{dR}{dt}$;

- Aperiodic (or oscillation) dynamic element of the second order: $f(t)=\tau_{22}^2\dfrac{d^3R}{dt^3}+\dfrac{d^2R}{dt^2}+\dfrac{dR}{dt}$,

where: $\tau_1, \tau_{21}, \tau_{22}$ are time constants.

As shown in [20], the flow speed can be described by differential equations of higher order when rheological conversions are multistage.

PHYSICAL AND MATHEMATICAL MODELS OF HEAT ENERGY TRANSFER IN AN ISOLATED BODY

Let us assume that there is a solid body the surface of which is perfectly isolated, and its length is $L \gg D$ where D is diameter (Fig. 3). The bare butt of the body momentary joins the source with infinitely large heat capacity. Let us divide the length of the body by n conventional areas with thickness $\Delta x \to 0$. Let us assume that to each additional area Δx_i heat energy is transferred only when the previous one takes the source temperature.

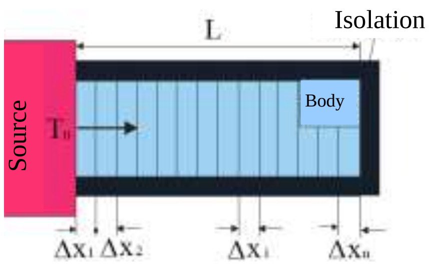

Fig. 3. Diagram of heat energy transfer in the rod with the isolated surface

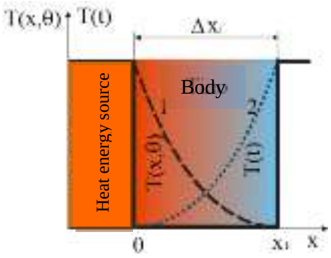

Fig. 4. IRT diagram and integrated impulse Dirac delta function for the isolated rod

In each element of the body there takes place the process of rheological conversion (e.g. heating), which can be described by the following equation:

$$\frac{\partial T_i(x,\theta)}{\partial\theta}=a\frac{\partial^2 T_i(x,\theta)}{\partial x^2}\quad(\theta>0,\,0<x<L),\qquad(31)$$

In the first part $\Delta x \to dx$ there is rheological heat energy transfer from the source to the first part of the body (Fig. 4, curve 1). Due to this fact the body part accumulates heat and is heated to temperature $T_{xi}=T_0$. The heating process of part $\Delta x_1 \to dx_1$ is shown in Fig. 4, curve 2. Integrated impulse Dirac delta function is a rectangle with Δx_1 width. Since according to the condition of the problem, the heat energy flow through the lateral surface is absent, then for each part Δx_i the problem of heat energy transfer and heating will be symmetrical.

Thus, the transfer process of the heat energy amount from the source to the rod part 1 will be described by the following differential equation:

$$\frac{\partial T_d(x,\theta)}{\partial\theta}=a\frac{\partial^2 T_d(x,\theta)}{\partial x^2},\ \ \theta>0,\ 0<x\le x_1,\qquad(29)$$

$$T_d(0,\theta)=T_{d0},\ T_d(x_1,0)=T_{x0},$$

where: T_{d0} is the source temperature, T_d is the current temperature in part Δx, corresponding to transferred heat energy.

The heating process of part Δx_i by the transferred heat energy will be described by such an equation:

$$\frac{\partial T_x(x,\theta)}{\partial\theta}=a\frac{\partial^2 T_x(x,\theta)}{\partial x^2},\ \ \theta>0,\ x_{i-2}<x_{i-1}\le x_i,\qquad(30)$$

$$T_x(0,\theta)=T_{x0},\ T_x(x_1,\theta)=T_{d0}.$$

In this case, the process of heat energy transfer along the rod may have the following variants:

1. The speed of heat energy transfer along the body is linear:

$$\gamma(t)=\frac{dT(t)}{dt}.\qquad(31)$$

2. The speed of heat energy transfer along the body is aperiodic of the first order:

$$\gamma(t)=\tau_1\frac{d^2T(t)}{dt^2}+\frac{dT(t)}{dt},\qquad(32)$$

where: τ_1 is the time constant of the speed of the heat energy flow.

3. The speed of heat energy transfer along the body is aperiodic of the second order:

$$\gamma(t)=\tau_{22}^2\frac{d^3T(t)}{dt^3}+\tau_{21}\frac{d^2T(t)}{dt^2}+\frac{dT(t)}{dt},\ \ \tau_{21}/\tau_{22}>2,\qquad(33)$$

where: τ_{21}, τ_{22} are time constants of the heat energy transfer speed.

4. The speed of heat energy transfer along the body is oscillatory:

$$\gamma(t)=\tau_{22}^2\frac{d^3T(t)}{dt^3}+\tau_{21}\frac{d^2T(t)}{dt^2}+\frac{dT(t)}{dt},\ \ \tau_{21}/\tau_{22}<2,\qquad(33)$$

If the speed of the heat energy flow is described by equation (31), then (30) takes the form:

$$\frac{\partial T_x(x,\theta)}{\partial\theta}+a\frac{\partial^2 T_x(x,\theta)}{\partial x^2}=-\frac{dT_x(t)}{dt}.\qquad(34)$$

Equation (34) is non-linear, since it contains such variables as: time θ of rheological heat energy transfer; time t of heat energy transfer along the body; the linear coordinate x, in which heat energy transfer takes place;

the IRT temperature $T_x(x,\theta)$ and the temperature $T_x(t)$ of heat energy transfer along the rod. For analytical solution of equation (34) we use the method of zero gradient. Under this method, if the temperature derivative regarding time is zero to the right and to the left of part Δx_1, then equation (34) is divided into the following system:

$$\frac{\partial T_x(x,\theta)}{\partial \theta}+a\frac{\partial^2 T_x(x,\theta)}{\partial x^2}=0; \qquad (35)$$

$$\frac{dT(t)}{dt}=T_x(x,\theta). \qquad (36)$$

The system of equations (35) and (36) is interconnected by the temperature $T_x(x,\theta)$. The solution of this system depends on conditions that apply to the process of heat energy transfer. If $\theta << t$, then firstly the temperature $T_x(x,\theta)$ is determined in equation (35), and then it is applied to equation (36). As mentioned above, the solution of equation (35) will be:

$$T_x(x,\theta)=T_0 erf\left(\Delta x_i/\sqrt{2a\theta}\right), \qquad (37)$$

where: T_0 is the temperature of the heat energy source, L is the body length.

Solution of equation (36) will be:

$$T(t)=t\cdot T_x(x,\theta). \qquad (38)$$

Applying (37) to equation (38), we get:

$$T(t)=T_0 t\cdot erf\left(\Delta x_i/\sqrt{2a\theta}\right). \qquad (39)$$

Provided that $\partial x^2=v^2\partial\theta^2$, equation (35) is driven to such a linear form:

$$\tau_\theta\frac{\partial T_x(x,\theta)}{\partial\theta}+T_x(x,\theta)=T_0, \qquad (40)$$

where $\tau_\theta=a/v^2$ is the time constant of rheological transition of the heat energy flow from the source to the IRT zone.

The solution of equation (40) will be:

$$T_x(x,\theta)=T_0\exp(-\theta/\tau_\theta), \qquad (41)$$

Applying (41) to equation (38), we have:

$$T(t)=T_0 t\cdot\exp(-\theta/\tau_\theta). \qquad (42)$$

Curves of transition processes of the body heating, calculated by formula (42) for values of heat energy θ transfer time, equal respectively: 0.5, 1.0, 1.5, 2.0 and 2.5, are shown in Fig. 5 (curves 1, 2, 3, 4 and 5).

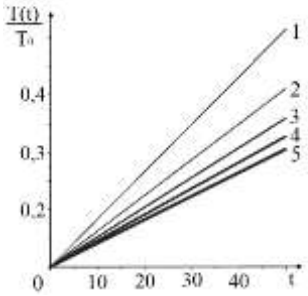

Fig. 5. Dynamic characteristics of the heat energy transfer process at its linear distribution

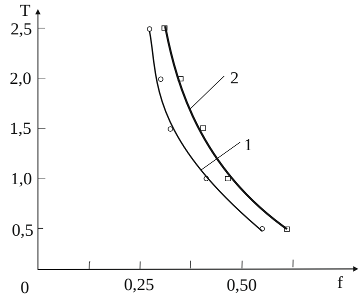

Fig. 6. Curves of models' adequacy at linear distribution of heat energy : 1 – change of function $erf\left(\Delta x_i/\sqrt{2a\theta}\right)$, 2 – change of function $\exp(-\theta/\tau_\theta)$

Fig. 6 shows the curves of models' adequacy at linear distribution of heat energy. Figure 6 shows that the difference between curves 1 and 2 is low, which suggests the adequacy of mathematical models. Equations (39) and (42) are mathematical models of the heat energy transfer process in the body at their linear speed. If $t=\infty$, then $\exp(-\theta/\tau_\theta)=0$, according to L'Hôpital's rule at $t=\infty$ $T_x(\infty)=T_0$, that is, the body heats up to the temperature of the heat energy flow. Let us consider transfer of the heat energy flow, when the flow speed is described by equation (32). Then (30) in view of equation (32) leads to the following nonlinear differential equation:

$$\frac{\partial T_x(x,\theta)}{\partial\theta}+a\frac{\partial^2 T_x(x,\theta)}{\partial x^2}=\tau_1\frac{d^2T(t)}{dt^2}+\frac{dT(t)}{dt}. \qquad (43)$$

According to the method of zero gradient, equation (43) is divided into the following system:

$$\frac{\partial T_x(x,\theta)}{\partial\theta}+a\frac{\partial^2 T_x(x,\theta)}{\partial x^2}=0, \qquad (44)$$

$$\tau_1\frac{dT(t)}{dt^2}+T(t)=T(x,\theta). \qquad (45)$$

where: $\tau_1=S/a$ is the time constant of the transfer process of the heat energy flow, S is the surface of heat energy transfer in the body during the flow.

At the initial conditions, equation (44) has the following solution:

$$T_x(x,\theta)=T_0 erf\left(x/\sqrt{2a\theta}\right), \qquad (46)$$

and equation (45):

$$T(t) = T_x(x,\theta)\left[1 - \exp(-t/\tau_1)\right], \qquad (47)$$

Applying (46) to equation (47), we obtain the mathematical model of such a process of the heat energy transfer flow in the following way:

$$T(t) = T_0 erf\left(x/\sqrt{2a\theta}\right)\left[1 - \exp(-t/\tau_1)\right]. \qquad (48)$$

The obtained mathematical model of the considered process is not very convenient for practical use, because it contains the function $erf(z)$ that needs to be expanded to series and limited by the number of its members, which, firstly, leads to long-lasting calculations, and secondly – accuracy of members becomes lower due to reduce in their amount. If condition $\partial x^2 = v^2 \partial\theta^2$ is executed, then we come to the following mathematical model:

$$T(t) = T_0 \exp(-\theta/\tau_\theta)\left[1 - \exp(-t/\tau_1)\right]. \qquad (49)$$

As shown by Acad. Lykov V.V., in the rod of great length there can be observed oscillatory processes of heat energy flow transfer. For this process, the nonlinear differential equation is:

$$\frac{\partial T_x(x,\theta)}{\partial\theta} + a\frac{\partial^2 T_x(x,\theta)}{\partial x^2} = \tau_{22}^2 \frac{d^3 T(t)}{dt^3} + \tau_{21}\frac{d^2 T(t)}{dt^2} + \frac{dT(t)}{dt^2}, \qquad (50)$$

where: $\tau_{21} = S/a$, $\tau_{22} = \sqrt{S/k}$ are time constants, S is cross-sectional area of the body, k is the heat conductivity coefficient.

The process of transferring of the heat energy flow will be oscillating when $\tau_{21}/\tau_{22} = \sqrt{Sk/a^2} < 2$. For the rod of round shape we have $\tau_{21}/\tau_{22} = R\sqrt{\pi k/a^2} < 2$. As for the given rod material the ratio is $k/a^2 = const$, then the ratio of time constants is completely determined by the rod radius. According to the method of zero gradient, equation (50) is divided into the following system:

$$\frac{\partial T_x(x,\theta)}{\partial\theta} + a\frac{\partial^2 T_x(x,\theta)}{\partial x^2} = 0, \qquad (51)$$

$$\tau_{22}^2 \frac{d^2 T(t)}{dt^2} + \tau_{21}\frac{dT(t)}{dt} + T(t) = T_x(x,\theta). \qquad (52)$$

Under initial conditions the solution of equation (52) will be:

$$T(t) = T_x(x,\theta)\left\{1 - \exp(\alpha t)\left[\cos(\omega_0 t) + (\alpha/\omega_0)\sin(\omega_0 t)\right]\right\}, \qquad (53)$$

where: $\alpha = -\tau_{21}/2\tau_{22}^2$ is the degree of oscillations damping, $\omega_0 = \sqrt{1/\tau_{22}^2 - \left(\tau_{12}/2\tau_{22}^2\right)^2}$ is natural frequency of oscillations.

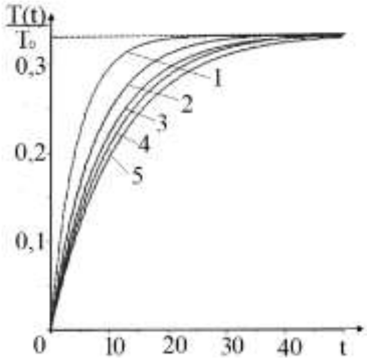

Fig. 7. Aperiodic dynamic characteristics of the heat energy transfer process

If the condition $\partial x^2 = v^2 \partial\theta^2$ is executed, then we obtain the following mathematical model of the considered process:

$$T(t) = T_0 \exp(-\theta/\tau_\theta)\left\{1 - \exp(\alpha t)\left[\cos(\omega_0 t) + (\alpha/\omega_0)\sin(\omega_0 t)\right]\right\}. \qquad (54)$$

Curves of transient processes of the body heating, calculated by formula (54) for time values of heat energy θ transfer, equal, respectively: 0.5, 1.0, 1.5, 2.0 and 2.5 are shown in Fig. 8 (curves 1, 2, 3, 4 and 5).

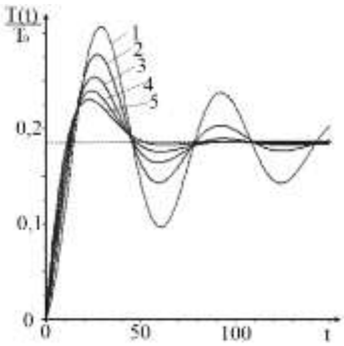

Fig. 8. Oscillation dynamic characteristics of the heat energy transfer process

CONCLUSIONS

1. It is shown that in the wall-adjacent layer, called the rheological transition zone, there are rheological conversions of heat energy transferred from the source. From the rheological conversion zone to the other fluid medium, there deduces (or flows) the result of this process, which may be heat energy, momentum or substance weight.

2. It is shown that in the rheological transition zone the process of heat energy transfer takes place under Fourier's or Newton's laws, and in the fluid medium there is the process of accumulation of energy, mass or momentum.

3. It is proposed to present the rheological process of heat energy transfer as impulse integral Dirac delta function with the core in the form of the temperature change, which is described by the nonlinear differential equation deduced based on the heat balance of the heat energy amount given by the source which deduces

(flows) to some fluid medium which may be liquid, gaseous, solid or other, such as viscoplastic.

4. If to present the rheological transition zone of heat energy transfer as integrated impulse Dirac delta function, on the border of which the derivative of the thermophysical parameter according to the time of heat energy transfer or according to distance is zero, the nonlinear differential equation is divided into the system of two interconnected linear differential equations, allowing to get analytical description of the considered non-linear process.

5. Based on the theory of rheological heat transfer (thermorheological effect), there have been considered the process of the body (fluid) heating according to the linear, periodic and oscillating accumulation of heat energy.

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ИССЛЕДОВАНИЕ ПРОЦЕССОВ ПЕРЕНОСА ТЕПЛОВОЙ ЭНЕРГИИ НА ОСНОВЕ ТЕОРИИ РЕОЛОГИЧНЫХ ПЕРЕХОДОВ И МЕТОДА НУЛЕВОГО ГРАДИЕНТА

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Аннотация. Показано, что перенос тепловой энергии от источника до среды сопровождается реологическими переходами. За счет переноса потока тепловой энергии с соответствующей скоростью в зоне реологического перехода происходит изменение физических параметров среды. Показано, что использование линейных градиентных законов при описании процессов переноса тепловой энергии приводит к большим расхождениям между теоретическими и экспериментальными результатами, а также к парадокса неограниченой скорости распространения возмущений температурных полей. Для математического описания процессов переноса тепловой энергии в средах предлагается использовать метод необратимых реологических переходов и нулевого градиента, что позволяет получать решения нелинейных дифференциальных уравнений в аналитической форме.

Ключевые слова: реологический переход; многофазная среда; теплопередача; моделирование.